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Innovation approach based measurement error self-correction in dynamic systems

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Abstract

In this work on the basis of the statistics of the relation of selection and theoretical variances, the simultaneous operative testing of mathematical expectation and variance of errors in the one-dimensional measurements are investigated.

A new structure and algorithmic provision of continuously operated measurement system with the error self-correction are presented. Mentioned measurement system allows to detect the changes in the statistical characteristics of errors and to carry out their correction on the basis of statistical analysis of Kalman filter innovation sequence in real time.

The proposed measurement systems structure with the error self-correction is simpler, as it does not require a model for the errors as that proposed in the existing literature.

Keywords: Measurement system; Error self-correction; Kalman filter; Innovation sequence, Statistical analysis

1. Introduction

As known, the metrological characteristics of measuring apparatus in the process of their operation do not remain stable. The dynamics of change of errors are in detail investigated and presented in [1].

The measuring apparatus utilized in the contents of the process control systems are subject to periodic inspection. Under the conditions of continuous production this creates the definite difficulties, connected with the stoppage of technological processes. In relation with this the development of methods of checking metrological characteristics of measuring apparatus without the stoppage of technological processes is an actual problem and is of undoubted practical interest.

To a solution of the problem, the work of the author [2] was dedicated. In this work on the basis of the statistics of the relation of two quadratic forms, matrices of which are the theoretical and selection covariance matrices of the normalized innovation sequence of Kalman filter, the operative method of checking the covariance matrices of the multidimensional measurement errors is proposed. In the mentioned work the problem of checking the mathematical expectation of measurement errors was not examined, which did not give the possibility to carry out complete testing of the statistical characteristics of errors. Furthermore, in this work

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any measurement error compensation procedure is not included.

In this work, measurement bias error compensation methods based on the bias error mathematical models are taken into consideration. For example, in [3,4] the logistic regression models and in [5] the time series model are used as an additive measurement bias error model. Adjustments to actual process measurements are then based on the estimates of bias error from the model. However, adequacy of mentioned mathematical models of measurement bias error has not been confirmed.

In this work on the basis of the statistics of the relation of selection and theoretical variances, the simultaneous operative testing of mathematical expectation and variance of errors in the one-dimensional measurements are investigated and measurement errors compensation method is proposed. This method is simpler, as it does not require a model for the errors as that proposed in [3–5].

2. Innovation sequence and its basic properties

Let us examine the linear one-dimensional dynamic system, represented by equations of state

$$x(k) = ax(k-1) + w(k) \tag{1}$$

and measurement

$$z(k) = x(k) + \xi(k), \tag{2}$$

where a is the known parameter; $\{w(k)\}$ is the system noise with the distribution $N(0, \sigma_w^2)$; $\{\xi(k)\}$ is the measurements noise (independent from the $\{w(k)\}$) with the distribution $N(0, \sigma_\xi^2)$; $x(0)$ is the initial condition.

In the well-investigated case of the Gaussian distribution of the value of $x(0)$, the minimum to the standard deviation $E[x(k) - \hat{x}(k)]^2$ (E the operator of statistical averaging) of estimation $\hat{x}(k)$ from the state of $x(k)$ is reached by the linear optimum Kalman filter [6]

$$\hat{x}(k) = a\hat{x}(k-1) + \frac{a^2P(k-1) + \sigma_w^2}{a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2}[z(k) - a\hat{x}(k-1)], \tag{3}$$

$$P(k) = a^2P(k-1) + \sigma_w^2 - \frac{\left[a^2P(k-1) + \sigma_w^2\right]^2}{a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2} = \frac{R\left[a^2P(k-1) + \sigma_w^2\right]}{a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2}, \tag{4}$$

where $P(k)$ is the variance of estimation errors. With the correspondence of the taken and actual statistical characteristics of the measurement errors, the innovation sequence

$$\Delta(k) = z(k) - a\hat{x}(k-1)$$

is white Gaussian noise with the zero mean and the variance $a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2$ [7,8], i.e.

$$\Delta(k) \sim N\left\lfloor 0, a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2 \right\rfloor. \tag{5}$$

For the purpose of checking the statistical characteristics of measurement errors more conveniently, the normalized innovation sequence is to use:

$$\widetilde{\Delta}(k) = \frac{\Delta(k)}{\sqrt{a^2P(k-1) + \sigma_w^2 + \sigma_\xi^2}}, \tag{6}$$

because in this case $\widetilde{\Delta}(k) \sim N(0,1)$.

Abnormal measurements, sudden shifts, which appear in the measurement channel, and also such faults as reduction of instrument accuracy, increase of noise background of instruments, etc., will cause changes of statistical characteristics of normalized innovation sequence (6). During the operation of process control systems the detection of such changes in real time, for the purpose of the subsequent correction of estimations or timely making decision about replacement of the measuring equipment, is required. Consequently, the problem of checking the metrological characteristics of measuring apparatus in this case yields to the fastest detection of the fact of difference of the characteristics from there nominal values.

3. Checking of the one-dimensional innovation sequence

For the purpose of testing mathematical expectation and variance of the one-dimensional innovation sequence $\widetilde{\Delta}(k)$ it is proposed to use the statistics

$$v(k) = \frac{(M-1)\tilde{s}^2(k)}{\sigma^2}, \tag{7}$$

depending on the relation of the selection and theoretical variances of the mentioned sequence, where

$$\tilde{s}^2 = \frac{1}{M-1}\sum_{j=k-M+1}^{k}\left[\widetilde{\Delta}(j) - \overline{\widetilde{\Delta}}(k)\right]^2 \tag{8}$$

and

$$\overline{\widetilde{\Delta}}(k) = \frac{1}{M}\sum_{j=k-M+1}^{k}\widetilde{\Delta}(j) \tag{9}$$