Mechatronics 32 (2015) 22–33

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

Optimal torque control of permanent magnet synchronous machines using magnetic equivalent circuits

Wolfgang Kemmetmüller*, David Faustner, Andreas Kugi

Automation and Control Institute, Vienna University of Technology, Gusshausstr. 27–29, Vienna, Austria

ARTICLE INFO

Article history:
Received 23 April 2015
Accepted 8 October 2015
Available online 14 November 2015

Keywords:
Optimal torque control
Magnetic equivalent circuit
Permanent magnet synchronous motor
Flatness based control

ABSTRACT

In recent years, permanent magnet synchronous machines (PSMs) are often designed in a mechatronic way to obtain e.g. special torque characteristics at zero currents or maximum efficiency. These designs are often characterized by a pronounced magnetic saturation and non-sinusoidal properties. This paper describes the optimal torque control of such PSMs utilizing a magnetic equivalent circuit (MEC) model. In contrast to approaches based on fundamental wave models (dq0-models), which utilize the Blondel–Park transformation and typically consider saturation and non-sinusoidal characteristics only in a heuristic way, MEC models allow to systematically account for these effects. Given the MEC model, optimal values for the coil currents are obtained from a constrained, nonlinear optimization problem, which can be efficiently solved by exploiting the special mathematical structure of the model. The results of the optimization are used in a flatness-based torque control strategy. The performance and practical feasibility of the proposed torque control concept are demonstrated by experiments on a test stand. Finally, it is shown that using this torque control in an outer angular speed control loop also proves to be beneficial.

1. Introduction

The accurate control of the torque is essential in many applications of permanent magnet synchronous machines (PSMs), which makes this topic an active field of research in recent years. The industrial standard to control PSMs is field oriented control (FOC), which is based on a fundamental wave model and the application of the Blondel–Park transformation, see, e.g., [1,2]. A number of research papers have discussed the development of advanced (nonlinear) control strategies based on this model. E.g., authors in [3–5] propose exact feedback linearization, authors in [6–8] use backstepping control and passivity based methods are applied in [9–11] to the control of PSMs. Furthermore, sliding mode control is examined in [12–14], model predictive control is used in [15–17] and direct torque control concepts can be found in [18–20]. These control strategies in general exhibit a good performance for PSMs and operating regions, which can be accurately described by a (magnetically linear) fundamental wave model (dq0-model).

For applications with high demands on torque, speed or position accuracy, it happens more often in recent years that motor designs are employed which do not satisfy the assumptions that have to be made for the derivation of classical dq0-models. In particular, fractional slot concentrated windings and rotors with interior permanent magnets are preferred by industry due to the simpler and cheaper construction. Moreover, to shape the torque characteristics especially for zero currents, inhomogeneous air gap geometries are frequently used in a mechatronic design approach. These constructions often yield pronounced nonsinusoidal (non-fundamental wave) characteristics of the back-emf and the inductances of the motor. Moreover, PSMs are often operated in a region, where significant saturation of the iron parts occurs.

Heuristic extensions of the dq0-model are typically proposed in literature to account for saturation and non-fundamental wave characteristics. E.g., the control strategies in [21–31] are based on an extension of the dq0-model by higher harmonics in the back emf, the inductances or the resulting torque. In these works, however, the influence of saturation and the resulting cogging or reluctance torque are not considered. Saturation is again incorporated into the dq0-model in a heuristic manner, see, e.g., [32–37].

The limitations of these approaches clearly result from the underlying dq0-model such that a more rigorous modeling approach is preferable. A magnetic equivalent circuit approach was described in [38,39] for the modeling of PSMs with internal or surface mounted magnets. It was shown that an accurate description of the behavior of PSMs with non-fundamental wave characteristics and significant saturation can be achieved with this approach. Since the resulting models feature a limited model complexity, these models can be considered a good basis for the controller design.

* Corresponding author. Tel.: +43 1 58801 37615; fax: +43 1 58801 37699.
E-mail addresses: kemmetmueller@acin.tuwien.ac.at (W. Kemmetmüller), faustner@acin.tuwien.ac.at (D. Faustner), kugi@acin.tuwien.ac.at (A. Kugi).

http://dx.doi.org/10.1016/j.mechatronics.2015.10.007
0957-4158/

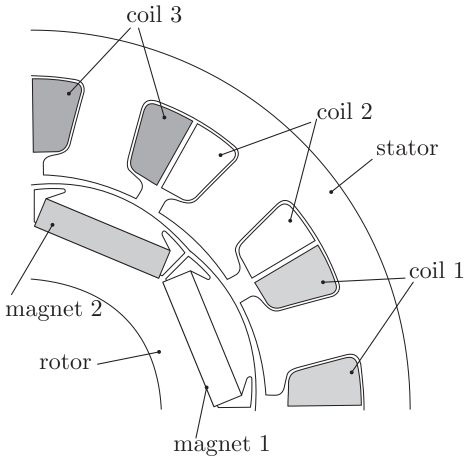

Fig. 1. Cross section of the considered PSM [38].

In this work, the optimal torque control of PSMs which shows both significant magnetic saturation and non-sinusoidal characteristics is considered. As a test case, a PSM with interior permanent magnets is used, which was designed for a rear steering system of a car. In this application, a motion of the PSM has to be prevented in case of a failure of the power electronics. This is achieved by designing an inhomogeneous air gap geometry which results in a large cogging torque. This in turn prevents undesired rotation of the motor at zero currents. However, the significant non-sinusoidal behavior and saturation complicates the accurate torque control of such PSMs. The corresponding mathematical model is described in [38], which will serve as a basis for the controller design. The control strategy is based on the solution of a nonlinear, constrained optimization problem in combination with a flatness-based feedback control. In [40], also the optimal torque control of a PSM, which exhibits significant saturation, based on an MEC model is considered. For this surface magnet PSM it is, however, possible to accurately approximate the characteristic quantities like the flux linkage of the coils by means of fundamental wave components, with only their amplitudes and phase angles being nonlinear functions of the coil currents. It is demonstrated in [40] how the fundamental wave characteristics can be beneficially utilized to solve the resulting optimal control problem. The present work deals with the more general case containing both magnetic saturation and non-fundamental wave characteristics.

The paper is organized as follows: The mathematical model in [38] is briefly summarized in Section 2. In Section 3, the calculation of optimal currents is described, which is used in the flatness based control strategy outlined in Section 4. Measurement results of a test stand are presented in Section 5 which demonstrate the good control performance and the practical feasibility of the proposed control strategy. Finally, Section 6 elaborates the benefits of using the optimal torque control strategy in an outer control loop for the angular speed.

2. Mathematical model

In [38], a general framework for the mathematical modeling of permanent magnet synchronous machines based on a magnetic equivalent circuit approach was derived. This approach was successfully applied to the modeling of both a surface-mounted PSM [39] and a PSM with internal magnets [38]. In this work, the same motor as in [38] will be used and therefore, the mathematical model derived in [38] serves as the basis for the development of the optimal torque control strategy.

The considered PSM with internal magnets comprises 12 coils and eight NdFeB-magnets. Fig. 1 depicts the cross section of a quarter of the motor. As already briefly discussed in the introduction, the PSM is used in an automotive application, where it is absolutely important that no motion occurs in the case of e.g. a failure of the power electronics. This behavior is achieved by designing an inhomogeneous air gap which yields a large cogging torque, see Fig. 1. Moreover, this design also results in a non-sinusoidal back emf and a significant influence of saturation in the stator and rotor. As shown in [38], the motor can be accurately described by an (magnetically nonlinear) MEC, comprising magneto-motive force (mmf) sources describing the coils and the permanent magnets, and magnetically nonlinear or position dependent permeances describing the stator, the rotor, the air gap and the leakages. The mathematical equations of this MEC are derived using network theory, well established in the modeling of electric networks, see, e.g., [41–43]. For this purpose, a tree being composed of elements of the MEC and connecting all nodes of the network without forming a mesh is defined. The choice of this tree is arbitrary except for the fact that all mmf sources of the MEC have to be part of the tree. The remaining elements of the network form the corresponding co-tree of the network. The interconnection of the tree and co-tree elements, i.e. the topology of the MEC, is described by the incidence matrix $\bar{\mathbf{D}}^T = \left[\bar{\mathbf{D}}_c^T, \mathbf{D}_m^T, \mathbf{D}_g^T\right]$, where $\bar{\mathbf{D}}_c = \mathbf{N}_c\mathbf{D}_c$, with the winding matrix $\mathbf{N}_c = \mathrm{diag}[N_c, N_c, N_c]$ and the number N_c of windings per coil. Therein, $\mathbf{D}_c$ is the part of the incidence matrix which is related to the coils, $\mathbf{D}_m$ is related to the permanent magnets and $\mathbf{D}_g$ is related to the permeances of the tree. The permeances of the tree and co-tree are combined in the (diagonal) permeance matrices $\mathbf{G}_t$ and $\mathbf{G}_c$, respectively. Both, $\mathbf{G}_t$ and $\mathbf{G}_c$, are nonlinear functions of the rotor angle and the corresponding mmfs. The mathematical model of the MEC can then be formulated in the form, see [38],

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_c^I = -R_c\mathbf{i}_c^I + \left(\bar{\mathbf{D}}_c^I\right)^T\mathbf{v}_c \tag{1a}$$

$$\mathbf{0} = \mathbf{K}\begin{bmatrix}\mathbf{i}_c^I\\ \mathbf{u}_{tg}\end{bmatrix} - \begin{bmatrix}\boldsymbol{\psi}_c^I\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\left(\bar{\mathbf{D}}_c^I\right)^T\bar{\mathbf{D}}_c\\ \mathbf{D}_g\end{bmatrix}\mathbf{G}_c\mathbf{D}_m^T\mathbf{u}_{tm} \tag{1b}$$

with

$$\mathbf{K} = \begin{bmatrix}(\bar{\mathbf{D}}_c^I)^T\bar{\mathbf{D}}_c\mathbf{G}_c\bar{\mathbf{D}}_c^T\mathbf{H}_1^I & (\bar{\mathbf{D}}_c^I)^T\bar{\mathbf{D}}_c\mathbf{G}_c\mathbf{D}_g^T\\ \mathbf{D}_g\mathbf{G}_c\bar{\mathbf{D}}_c^T\mathbf{H}_1^I & \mathbf{G}_t + \mathbf{D}_g\mathbf{G}_c\mathbf{D}_g^T\end{bmatrix} \tag{2}$$

where $\boldsymbol{\psi}_c^I$ is the vector of independent flux linkages, $\mathbf{i}_c^I$ is the corresponding vector of independent coil currents and R_c is the electrical resistance of a coil. The influence of the coil voltages $\mathbf{v}_c$ on the flux linkage is described by the matrix $\left(\bar{\mathbf{D}}_c^I\right)^T$, which reflects the magnetic connection of the coils. The set of algebraic Eq. (1b) describes the independent coil currents $\mathbf{i}_c^I$ and the mmfs $\mathbf{u}_{tg}$ of the permeances of the tree of the magnetic network as a function of the flux linkage $\boldsymbol{\psi}_c^I$ of the coils and the mmfs $\mathbf{u}_{tm}^T = [u_{ms}, -u_{ms}]$ of the permanent magnets, cf. [38]. Finally, $\mathbf{H}_1^I$ results from the inverse of a transformation matrix $\mathbf{T}_{1c}$, which has been used in [38] to eliminate the redundancies of the nonlinear algebraic equations, in the form $\mathbf{T}_{1c}^{-1} = [\mathbf{H}_1^{\perp}, \mathbf{H}_1^I]$.

The torque produced by the motor is given by

$$\tau = \frac{1}{2}p\left(\mathbf{u}_{tg}^T\frac{\partial\mathbf{G}_t}{\partial\varphi}\mathbf{u}_{tg} + \left[(\mathbf{H}_1^I\mathbf{i}_c^I)^T, \mathbf{u}_{tm}^T, \mathbf{u}_{tg}^T\right]\bar{\mathbf{D}}\frac{\partial\mathbf{G}_c}{\partial\varphi}\bar{\mathbf{D}}^T\begin{bmatrix}\mathbf{H}_1^I\mathbf{i}_c^I\\ \mathbf{u}_{tm}\\ \mathbf{u}_{tg}\end{bmatrix}\right), \tag{3}$$

with $p = 4$ being the number of pole-pairs of the motor.

A detailed derivation of this model and an evaluation of the model accuracy is given in [38], where a slightly different notation is used. It should be noted that the optimal control strategy developed in this manuscript can be applied to any motor construction which can be described by an MEC model of the form (1)–(3).

3. Calculation of optimal coil currents

The main goal of this work is to derive a control strategy which calculates the control inputs $\mathbf{v}_c$ in a way that the torque τ tracks a