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Mechatronic design of a self-balancing three-dimensional inertia wheel pendulum

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ABSTRACT

In this paper a three-dimensional self-balancing cube is introduced. For stabilization and swing-up, reaction wheels are placed within the cube along with the actuation, electronics and sensors. This allows the cube to swing up from a rest position and to balance around the unstable steady-state positions. We conducted a design assessment to review different design approaches. For the most promising design, we formulated a parameter optimization to calculate the most suitable dimensions for the cube and the inertia elements. These results were used in the mechanical design stage to obtain the optimal design parameters for the cube in an iterative process. To show the effectiveness of the test bench we considered a proportional control law for swing-up and stabilization at the various steady state positions.

1. Introduction

Various kinds of underactuated inverted pendulum, such as the Furuta pendulum or the inverted pendulum on a cart, are used as reference models in control theory. Another type of inverted pendulum is the two-dimensional inertia wheel pendulum [1–3], which consists of a physical pendulum and a rotating reaction wheel. We extended this two-dimensional design to a three dimensional (3D) pendulum that has the shape of a cube and is able to balance on its tip and edges. As in the 2D case, the pendulum uses reaction wheels to balance the cube around its equilibrium positions. Analogous to the two dimensional pendulum, the 3D pendulum we propose is hereafter referred to as the *inertia wheel cube* (IWC).

Due to their mechanical structure, the simple but non-linear dynamics of inertia wheel pendulums can be used as a benchmark for non-linear control algorithms. From a pedagogical point of view, inertia wheel pendulums can also serve as simple reference control problems to demonstrate basic ideas of control theory. When balancing on one of its edges, the cube behaves like a 2D inertia wheel. The entire 3D dynamics can be explored while the cube is balancing on its tip.

As already mentioned, inertia wheel pendulums exploit the effects of inertia wheels. Unlike gyroscopic flywheels [4], where a gimbal-suspensioned wheel rotates with a constant speed around its principal axis of inertia, inertia wheels, also often referred to as reaction wheels, rotate with a variable speed.

Various robotic systems using gyroscopic flywheels to stabilize a vehicle have been presented. While in [5] the stabilization of a bicycle was demonstrated, in [6,7] the authors successfully stabilized a single-wheel robot. Both systems use gimbal motors to deflect the rotating flywheel. Deflecting the rotating flywheel applies a torque around the axis perpendicular to the plane spanned by the axis of deflection and the axis of rotation of the flywheel to the system. By deflecting the flywheel around the axis of gravity, this torque can be used to stabilize the vehicle around the unstable position of rest. Another robotic system consisting of a remote-controlled vehicle and a gyroscopic flywheel was proposed in [8]: the wheel rotates with constant speed and allows driving curves while the car crosses over to the inside of the curve against the centrifugal forces.

Reaction wheels are typically used, for instance, for attitude control in spacecrafts and satellites. Usually, three reaction wheels are mounted along three perpendicular axes. In some cases a fourth inertia wheel is added for redundancy. By varying the speed of the reaction wheels, a reaction torque is applied to the supporting structure. In the case of a free-floating system, such as a satellite or a spacecraft, this reaction torque leads to a rotation of the supporting structure.

Clearly, both kinds of flywheel are able to stabilize an inverted pendulum, but they use different physical effects. To stabilize our 3D inverted pendulum, we employed an approach similar to that used in satellites in our work.

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Two robots using reaction wheels for stabilization have recently be presented by Murata and can be found on their web page [9]. While Murata Boy is able to ride a bicycle Murata Girl rides on a unicycle. Both robots are stabilized by reaction wheels.

The authors of [10] proposed a testbed for spacecrafts. The testbed can either be actuated by reaction wheels, fan thrusters or by moving prove masses. The first actuation approach is thus similar to the idea utilized in this paper.

A similar project, named Cubli [11,12], has recently been realized. This system utilizes three inertia wheels mounted on three perpendicular axes. In contrast to the stabilization of satellites, this approach uses active breaks to decelerate the reaction wheels and to swing up the cube to its various steady-state positions. This particular swing-up strategy has the drawback that actively controlling the cube during the brake phase is not possible. As soon as the brakes are released, the motor torques can again be used to control the cube.

In [13] the authors proposed an other three dimensional self balancing cube. Different to our approach this cube does not use flywheels for stabilization. Instead six arms are used to stabilize the cube at its unstable equilibrium positions by rotating the arms and thus shifting the center of mass of the cube.

In this paper, we present a cube that employs a similar actuation system as Cubli, but is able to actively control the swing-up phase, as no breaking system is used. One advantage over the system used in the Cubli test bench is that our system is particularly useful for visualizing gyroscopic effects. One disadvantage compared to Cubli, however, is that it is not capable of swinging up from arbitrary positions to arbitrary unstable positions, as the power-to-weight-ratio of the electric motors is not sufficiently high to generate the reaction torque necessary.

Since our system exhibits non-linear dynamics, it can be used as a test bench for non-linear control algorithms [14,15]. In [16], non-linear control approaches for the inverted pendulum on a cart were introduced. Various non-linear control approaches have been presented that successfully control the attitude of satellites in space. In [3,2,17] various non-linear regulators were applied to the 2D inertia wheel pendulum. A non-linear control approach employing integrator backstepping was used in [18] to control the Cubli. In this paper we are mainly interested in the mechatronics design approach of such a device, but to show the effectiveness of the proposed test bench, we additionally implemented a first approach of a balancing control.

This paper is structured as follows. Before describing the mechatronics design of the IWC in Section 2, we evaluate and compare various design concepts in terms of expected system performance and necessary design efforts. In Section 3, a detailed kinematic and dynamic model is derived for the 3D inverted pendulum and then used in Section 4 to calculate the optimal size of the cube. In Section 5 the mechanical and electrical design of the cube is described next to a linear control approach used to evaluate the test bench. Sections 6 present simulation and experimental results, respectively, and Section 7 concludes the paper.

2. Design survey

Before we start on the mechatronics design of the IWC, we evaluate and compare various design concepts in terms of system performance and design effort, in order to select a suitable solution to the given task. In total, three different designs are considered, which are summarized in Table 1.

The first design involves a hollow sphere which is supported and driven by three perpendicular omni directional friction wheels that can rotate the sphere in any direction. In this case, the hollow sphere acts as a 3D inertia wheel. The second and third concepts both use conventional cylindrical inertia wheels. While the former employs a separate inertia wheel on every face of the cube, the latter utilizes only three inertia wheels on the faces next to the tip on which the cube is balanced.

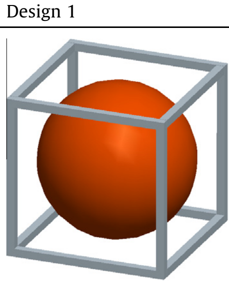
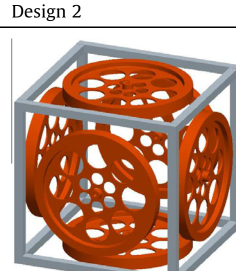
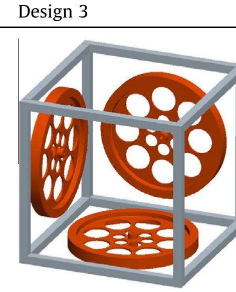
Neglecting the reaction wheel dynamics, the mass of the cube frame and the actuation system, we obtain the simplified model in Fig. 1. This model reduction yields the following equations of motion in the 2D case

$$\tau = (J_0 - J_w)\ddot{\varphi} + mga(\varphi), \tag{1}$$

where φ is the position of the cube, τ is the motor torque, J_w is the inertia of the reaction wheel around its axis of rotation, J_0 is the inertia of the whole cube around the edge of the cube, m is the total mass of all reaction wheels, g represents the gravity constant and $a(\varphi)$ is the distance of the center of mass to the edge. Assuming a constant mass m of the reaction wheels, the same side lengths L of the cubes and the same actuation system (represented by similar maximal torques τ_{max}) for all three designs, allows performance comparison on the basis of the simplified 2D model. For $\dot{\varphi}(t) = 0$ the first term on the right hand side of Eq. (1) vanishes and the necessary torque to hold the cube at a certain position is given by $\tau_{hold} = mga(\varphi)$. As m and g are independent of the chosen design variant thus the design which minimizes $a(\varphi)$ is the favorable design. The values for $a(\varphi)$ for $\varphi = 0$ are summarized at the bottom of Table 1.

It can be seen that the last concept with only three cylindrical inertia wheels achieves the highest acceleration to swing up the cube from its resting position. Furthermore, this design variant also seems to involve the least mechanical design effort as mounting and actuation of the inertia elements are straightforward compared to the variant with the hollow sphere.

Table 1
Design variants considered.

Design 1	Design 2	Design 3
		
$a(0) = \frac{L}{2}$	$a(0) = \frac{L}{2}$	$a(0) = \frac{L}{3}$