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Technical note

Presliding hysteresis damping of LuGre and Maxwell-slip friction models

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ABSTRACT

In this paper, the damping properties of presliding hysteresis are explored for the LuGre and Maxwell-slip friction models. Taking out of consideration the classical linear viscous damping and Stribeck effect, the nonlinear damping of force–displacement hysteresis is analyzed, in Lyapunov sense, for the motion dynamics. Based thereupon the simple but straightforward criteria of model's parametrization are derived for kinetic friction to be dissipative. Further we show a related experimental example of presliding hysteresis friction response in vicinity to zero velocity.

1. Introduction

The significant property of kinetic friction, which is known to be crucial for controlled motion systems, is the presliding hysteresis in displacement. That appears as the nonlinear force transitions with memory each time the relative motion changes its direction or gets started from an initial state. For the review of nonlinear friction dynamics, including presliding hysteresis, see [2] and references therein. Several dynamic friction models capture the presliding hysteresis behavior in different ways, at that point providing the non-local or local memory (see e.g. [18] for details), drifting or non-drifting, rate-independent or rate-dependent hysteresis, and other friction-related properties. For overview and classification of the presliding behavior captured by different dynamic friction models see [6]. Furthermore, a theoretical analysis of dynamic system behavior with hysteresis element in the feedback – the case similar to that addressed in this work – has been consider in [3], by the approximate describing function method and its comparison with numerical simulations.

In this work we consider the LuGre [9,7] and Maxwell-slip [15,10,17] dynamic friction models as two of the most widespread approaches which cardinally differ in their manner of describing the hysteresis force transitions. Note that other dynamic friction models can be equally considered, see e.g. [6] for reference. However, our analysis is limited to the two above models as these are particularly suitable and often used for applications like control, real-time simulation, diagnosis and monitoring and others.

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The aim of this contribution is to analyze the damping characteristics of presliding hysteresis and that for the free motion dynamics, with displacement y, of the type

$$\ddot{y} = -F(\dot{y}), \tag{1}$$

where F is captured by either LuGre or Maxwell-slip friction model. Note that the unity mass is assumed in this work for the sake of simplicity. Since we deliberately concentrate on the hysteresis damping of presliding friction, the more classical linear viscous friction damping is taken out of consideration. Also no velocity-weakening friction mechanisms like the well-known Stribeck effect are considered. Here it is worth to recall that in particularly the Stribeck effect has been put in perspective when analyzing the dissipative behavior of nonlinear friction and corresponding stability of its control, see [5,8,12,16]. On the contrary, in this study we are interested in emphasizing the hysteresis-related friction damping since we believe that at motion reversals and motion stop, which are most significant for such applications as precise positioning control, these type of friction effects are pivotal and should be better understood for modeling. The analysis, accomplished in the following, should help to understand better the parameterization of LuGre and Maxwell-slip friction models when these are used for describing the motion dynamics. The demonstrated, also with an experimental example, damping behavior of a free motion close to a motion stop or reversal provides further insight into the nonlinear system dynamics i.e. at a position settling. Below, we briefly summarize the LuGre and Maxwell-slip friction models with notice on what the following main results aim to analyze. After the main results, we look on an experimental example of nonlinearly-damped motion in vicinity to stop, that argues in favor of the analysis made.

1.1. LuGre model

The LuGre friction model, first introduced in [9] and revisited in [7], is based on the so-called bristle approximation of two friction surfaces in contact, see Fig. 1 (cf. with Fig. 5 in [11] and Fig. 1 in [9]). The associated tangential friction force, which is acting in opposite direction to the relative velocity $\dot{y}$, is described by

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{y}, \tag{2}$$

and constitutes a linear combination of internal friction state z, its damping, and viscous friction term which is linear to the relative velocity. The positive parameter σ_0 is the stiffness of asperity contacts and the non-negative parameters[1] σ_1 and σ_2 determine respectively the internal state damping, associated with a micro-displacement, and velocity-dependent damping of the viscous friction. The internal friction state is governed by

$$\dot{z} = \dot{y} - \sigma_0 \frac{|\dot{y}|}{g(\dot{y})} z, \tag{3}$$

where the positive nonlinear static map

$$g(\dot{y}) = F_c + (F_s - F_c) \exp\left(-(|\dot{y}|/V_s)^{\mu}\right), \tag{4}$$

captures the so-called Stribeck effect. The parameters $F_c > 0$ and $F_s \geqslant F_c$ are correspondingly the Coulomb and stiction friction coefficients. The exponential parameters V_s and μ are the Stribeck velocity of the convergence of g towards F_c and Stribeck shape factor[2] correspondingly.

Regarding the dissipative nature of friction, the fundamental property of a passive operator $\dot{y} \mapsto F$ has been investigated in [8] and later in [7]. In [7] it has been formulated the I/O dissipativity of the LuGre friction model as a condition of the parameter inequality (see Eq. (7) of Property 3 in [7])

$$\sigma_2 > \sigma_1 (F_s - F_c)/F_c. \tag{5}$$

While giving the necessary and sufficient conditions of passivity of friction operator the '*if and only if*' condition in [8] has been formulated as (cf. with Eq. (2.4) in [8])

$$1/F_c \leqslant 1/g(\cdot)(1 + \sigma_2/\sigma_1). \tag{6}$$

It is worth noting that the condition (6), when assuming $g = F_c$, results in

$$\sigma_2/\sigma_1 \geqslant 0, \tag{7}$$

thus allowing for $\sigma_2 = 0$. Recall that for analysis accomplished in this work we assume both $g = F_c$ and $\sigma_2 = 0$. On the contrary, the parameter inequality from [7] requires $\sigma_2 > 0$ and thus constitutes a weaker condition which does not allow excluding the viscous friction damping.

1.2. Maxwell-slip model

The Maxwell-slip friction model can be demonstrated by means of the associated Maxwell-slip structure depicted in Fig. 2(a). N parallel-connected elasto-plastic elements are either sticking or slipping while being connected to the common velocity input.

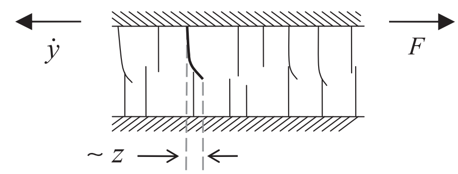

Fig. 1. Bristle approximation of two friction surfaces in contact. The tangential force F acts in opposite direction to velocity $\dot{y}$. The internal friction state z maps the average deflection of bristles.

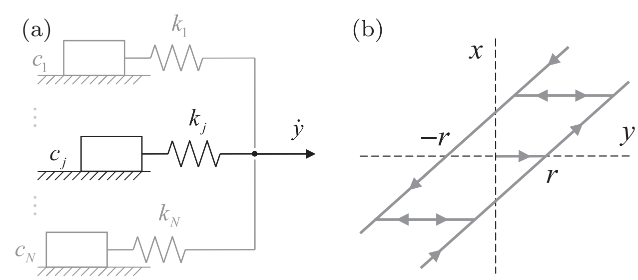

Fig. 2. Maxwell-slip structure (a), transfer characteristics of play-type (backlash) hysteresis operator (b).

Each element is parameterized by its individual stiffness k_j and breakaway force c_j, so that the element is sticking as long as $k_j|y_j| < c_j$. Afterwards, the element starts to slip until the next motion reversal. All the element blocks are assumed as massless so that no dynamic interaction occurs at relative displacement. The associated output friction force of each element is $F_j = k_j y_j$ when sticking and $F_j = c_j \,\mathrm{sign}(\dot{y})$ when slipping. Since the elements act in parallel the total friction force is given by

$$F(\dot{y}) = \sum_{j=1}^{N} F_j. \tag{8}$$

An important model feature is that each elasto-plastic Maxwell element can be described by means of the play- and stop-type hysteresis operators (see e.g. [14,13] for details). An elasto-plastic Maxwell element offers the same transfer characteristics as the stop-type hysteresis operator which in turn results from his play-type counterpart. The transfer characteristics of a play-type hysteresis operator are shown in Fig. 2(b). The operator provides a multi-valued rate-independent map $y \mapsto x$ depending on the initial state x_0 and parameter $r > 0$. The latter determines the width of the play-zone which is $2r$. The operator dynamics in the differential form is given by

$$\dot{x} = \begin{cases} \dot{y} & \text{if } x - r = y \ \vee \ x + r = y, \\ 0 & \text{if } x - r < y < x + r. \end{cases} \tag{9}$$

To that end, the stop-type hysteresis operator is transformed from its play-type counterpart by the simple algebraic relation $y - x(y, r)$. This allows constructing the single elasto-plastic Maxwell element j as

$$F_j = k_j \big(y - x(y, r_j)\big). \tag{10}$$

The corresponding parametrization requires $r_j = c_j/k_j$. We note that the Maxwell-slip friction model as in (8)–(10) is of a pure hysteresis nature and no relaxation related to the viscous damping or Stribeck effect should be done.

Now we are in a position to analyze the hysteresis damping behavior of each of the above friction models when assuming a free motion response starting from some initial state $\dot{y}(0) \neq 0,\ F(0) = F_c \,\mathrm{sign}(\dot{y}(0))$. Recall that during an unidirectional

[1] The original LuGre friction model assumes the damping parameters to be positive. In this work, however, we relax the positive definiteness and require both damping parameters to be non-negative.

[2] The original LuGre friction model and multiple works related to that assume $\mu = 2$. From other friction-related studies, it is known that μ Stribeck shape factor can assume various values, even negative, therefore providing an additional degree of freedom for capturing the static characteristic friction curve $g(\dot{y})$.