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Electromagnetic interaction force between two noncoaxial circular coils

Alon Benhaim, Osher Shapira, David Elata *

Technion – Israel institute of Technology, Haifa, Israel

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ABSTRACT

The electromagnetic interaction forces between two noncoaxial circular coils have been previously analyzed. In the present study we revisit that solution and rederive these forces in a new functional form which provides new insight. Specifically, we revisit the notion of a neutral plane, at a critical vertical separation, in which alignment forces identically vanish. To this end we present a new simplified 2D model of the problem, in which it is easier to understand the nature of these forces. We show that our simplified 2D model captures the same response characteristics as in the more complex 3D problem of the interaction between coils. We show the context and range of parameters in which a local neutral plane exists.

1. Introduction

The electromagnetic interaction force between two conducting coils is a fundamental problem which is relevant to electromagnetic actuation [1–6] and may also be relevant to alignment processes in assembly [7,8]. The aligning force between two non-coaxial circular rings has been investigated by Kim et al. [9]. In that paper, the two components of the interaction force – the aligning force (parallel to the rings plane) and the levitating force (normal to the rings plane), were analyzed for the case of two conducting rings of different diameters. It was suggested that at a critical vertical separation between the rings, the aligning force vanishes for any given eccentricity, constituting a neutral plane of zero aligning response. The physical nature of the interaction forces between two rings, and the nature of its aligning and levitating components, is somewhat difficult to comprehend. Specifically, the notion of a neutral aligning response seems nontrivial. This is not only because of the rather complex geometry, but also because of the elaborate functional form of the forces (it is not easy to deduce the nature of the interaction forces from these functional forms).

In this work we revisit the solution proposed by Kim et al. [9] and present it in a newer more compact functional form. We thoroughly investigate the aligning and levitating forces for different relative diameters, vertical separations, and different eccentricity.

To this end, we present a new simplified 2D model of the problem which provides new insight, and makes it easier to comprehend the effects of geometrical parameters that determine the electromagnetic forces. We present these forces in a new graphic form that maps-out positive/negative regions of the aligning and levitating forces. Specifically, we revisit the notion of a neutral plane, at a critical vertical separation, in which the aligning force identically vanishes. With our simplified model we show the context and range of parameters in which a local neutral plane does exist.

We then map the interaction forces for the original 3D problem of two conducting rings, and show the similarity of the two solutions [10].

2. The electromagnetic forces between two conducting rings

The two conducting rings considered by Kim et al. [9] are presented in Fig. 1a. The origin of the Cartesian coordinate system is located at the center of the bottom ring of radius ρ_B, and the center of the top ring of radius ρ_T is located at a vertical separation $z = Z$ and radial eccentricity $x = R$. The bottom and top rings conduct currents in the clockwise direction I_B and I_T, respectively. Kim et al. derived the interaction forces from the electromagnetic potential of the system. Here we rederive the same equations by considering the forces applied to the top conducting ring by the magnetic field which is induced by the current in the bottom ring.

According to the Bio-Savart law, the magnetic flux density produced by the bottom ring at a general point in space (x, y, z) (Fig. 1b) is given by

$$\mathbf{B}_B = \frac{\mu_0 I_B}{4\pi} \int \frac{d\mathbf{l}_B \times \mathbf{r}_B}{|\mathbf{r}_B|^2} \tag{1}$$

An infinitesimal vector in the direction of the current flowing in the bottom ring is given by

* Corresponding author. Tel.: +972 4 829 3184.
E-mail address: elata@technion.ac.il (D. Elata).

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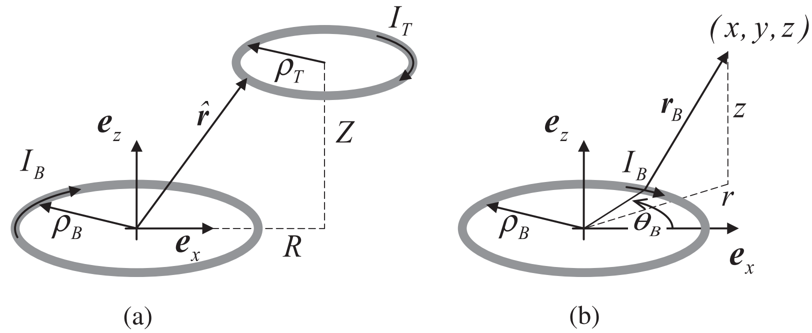

Fig. 1. (a) Two conducting rings with radii $\rho_B,\ \rho_T$ and currents $I_B,\ I_T$, respectively. (b) Schematic geometry for calculation of the magnetic flux density $\boldsymbol{B}$ produced by the bottom ring.

$$d\mathbf{l}_B = \rho_B d\theta_B \cdot [sin(\theta_B)\mathbf{e}_x - cos(\theta_B)\mathbf{e}_y] \tag{2}$$

The vector between the current in this segment to a general point in space (x, y, z) is given by

$$\mathbf{r}_B = [x - \rho_B cos(\theta_B)]\mathbf{e}_x + [y - \rho_B sin(\theta_B)]\mathbf{e}_y + z\mathbf{e}_z \tag{3}$$

Substituting (2) and (3) into (1), we obtain the magnetic flux density in the Cartesian coordinate system

$$\boldsymbol{B}_B = -\frac{\mu_0 I_B \rho_B}{4\pi}\int_0^{2\pi} \frac{z cos(\theta_B)\mathbf{e}_x + z sin(\theta_B)\mathbf{e}_y - [x cos(\theta_B) + y sin(\theta_B) - \rho_B]\mathbf{e}_z}{\{x^2 + y^2 + \rho_B^2 + z^2 - 2\rho_B[x cos(\theta_B) + y sin(\theta_B)]\}^{3/2}} d\theta_B \tag{4}$$

Expressing this flux density using the Cylindrical coordinates system $\boldsymbol{e}_r, \boldsymbol{e}_\phi, \boldsymbol{e}_z$ and developing mathematically, we obtain:

$$\boldsymbol{B}_B = -\frac{\mu_0 I_B \rho_B}{2\pi}\int_0^{\pi} \frac{z cos(\theta_B)\boldsymbol{e}_r + [\rho_B - r cos(\theta_B)]\boldsymbol{e}_z}{[r^2 + \rho_B^2 + z^2 - 2\rho_B r cos(\theta_B)]^{3/2}} d\theta_B \tag{5}$$

where $r = \sqrt{x^2 + y^2}$.

This is the magnetic flux density produced by the bottom ring, at a general point in space with radial eccentricity r and vertical separation z. In order to calculate the force acting between the rings we integrate

$$\boldsymbol{F} = I_T \int d\mathbf{l}_T \times \boldsymbol{B}_B \tag{6}$$

The infinitesimal vector in the direction of the current in the top ring is

$$d\mathbf{l}_T = \rho_T d\theta_T \cdot [\sin(\theta_T)\boldsymbol{e}_x - \cos(\theta_T)\boldsymbol{e}_y] \tag{7}$$

The vector between the center of the bottom ring to a segment of wire in the top ring is given by (Fig. 1a)

$$\hat{\boldsymbol{r}} = (R + \rho_T \cos(\theta_T))\boldsymbol{e}_r + \rho_T \sin(\theta_T)\boldsymbol{e}_y + Z\boldsymbol{e}_z \tag{8}$$

The radial and vertical components of the distance between a point on the top ring to a point on the bottom ring are

$$r = |\hat{\boldsymbol{r}} - Z\boldsymbol{e}_z| = \sqrt{R^2 + \rho_T^2 + 2\rho_T R \cos(\theta_T)}, \quad z = Z \tag{9}$$

Substituting (5), (7) and (9) into (6) we may write the force components in the following form

$$F_r = \frac{\mu_0 I_B I_T \rho_B \rho_T}{4\pi}\int_0^{2\pi}\int_0^{2\pi} \frac{[\rho_B - R\cos(\theta_B) - \rho_T \cos(\theta_B - \theta_T)]\cos(\theta_T)}{D^{3/2}} d\theta_B d\theta_T \tag{10}$$

$$F_z = -\frac{\mu_0 I_B I_T \rho_B \rho_T}{4\pi}\int_0^{2\pi}\int_0^{2\pi} \frac{Z\cos(\theta_B - \theta_T)}{D^{3/2}} d\theta_B d\theta_T \tag{11}$$

where $D = D_{con} + D_{ecc}$. The radial force component (10) is the aligning force, and the vertical force component (11) is the levitating force (it is quite trivial that F_ϕ vanishes mathematically due to symmetry).

The integrand denominator components are given by

$$D_{con} = \rho_T^2 + \rho_B^2 + Z^2 - 2\rho_B\rho_T \cos(\theta_B - \theta_T) \tag{12}$$

$$D_{ecc} = R(R + 2\rho_T cos(\theta_T) - 2\rho_B cos(\theta_B)) \tag{13}$$

The term D_{con} describes the denominator for no radial eccentricity (i.e. when the rings are concentric $D|_{R=0} = D_{con}$), and it is well known in electromagnetic calculations associated with concentric circular loops [11]. The term D_{con} is essentially the distance between a point on the bottom ring to a point on the top ring for concentric rings. The different locations of the points on each ring are described by the angles θ_B, θ_T. The term D_{ecc} describes the addition to the distance D_{con} when the rings are eccentric with radial eccentricity R.

The terms for the aligning and levitating forces given by (10) and (11), are compatible with Kim's results (Eqs. (23) and (25) in [9]). The expressions we present here are simpler in form and provide more insight. In our new formulation it is easier to identify the effect of geometry on the different terms in the integrands.

The result demonstrates that the force depends on the radial eccentricity R, vertical separation Z, the radii and the currents

$$F_{r\backslash z} = F_{r\backslash z}(I_B, I_T, \rho_B, \rho_T, R, Z) \tag{14}$$

It is constructive to consider the system in terms of non-dimensional variables

$$\widetilde{F}_{r\backslash z} = \frac{\pi}{4\mu_0 I_B I_T} F_{r\backslash z}, \quad \widetilde{R} = \frac{R}{\rho_B + \rho_T}, \quad \widetilde{Z} = \frac{Z}{\rho_B + \rho_T} \tag{15}$$

Also it is convenient to define the ratio between the radii as a non-dimensional variable, $\tilde{r} = \rho_T/\rho_B$. In terms of these non-dimensional variables the aligning and levitating non-dimensional forces may be rewritten in the following form

$$\widetilde{F}_r = \frac{\tilde{r}}{16}\int_0^{2\pi}\int_0^{2\pi} \frac{[1 - \widetilde{R}(1 + \tilde{r})\cos(\theta_B) - \tilde{r}\cos(\theta_B - \theta_T)]\cos(\theta_T)}{\widetilde{D}^{3/2}} d\theta_B d\theta_T \tag{16}$$

$$\widetilde{F}_z = -\frac{\tilde{r}}{16}\int_0^{2\pi}\int_0^{2\pi} \frac{\widetilde{Z}(1 + \tilde{r})\cos(\theta_B - \theta_T)}{\widetilde{D}^{3/2}} d\theta_B d\theta_T \tag{17}$$

where $\widetilde{D} = \widetilde{D}_{con} + \widetilde{D}_{ecc}$ and the non-dimensional integrand denominator components are given by

$$\widetilde{D}_{con} = [\tilde{r}^2 + 1 + (1 + \tilde{r})^2(\widetilde{Z}^2) - 2\tilde{r}\cos(\theta_B - \theta_T)] \tag{18}$$

$$\widetilde{D}_{\widetilde{R}} = (1 + \tilde{r})\widetilde{R}[(1 + \tilde{r})\widetilde{R} + 2(\tilde{r} cos(\theta_T) - cos(\theta_B))] \tag{19}$$

An interesting mathematical characteristic of the two rings configuration is derived from its symmetry. It is trivial that if we switch the position of the rings (rotating the x–y plane by 180