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Frequency estimation by iterative interpolation based on leakage compensation

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ABSTRACT

An iterative interpolation method for estimation parameters that characterize a linear combination of sinusoids is presented. It is based on the analysis of the generation principle of the spectrum leakage, picket fence effect and an interpolation scheme applied to the discrete Fourier transform, and is described as follows: (1) Estimate parameters by traditional interpolation method; (2) obtain the leakage compensation factors; (3) re-calculate parameters by interpolation method based on leakage compensation; Repeat (2) and (3) until obtain the high precision result. The simulation results show that the proposed iterative method has the better estimation accuracy and stability under noise and the severe long-range leakage situation, and has fast convergence rate. It can be used as an alternative method to improve parameter estimation accuracy of the discrete Fourier transform.

1. Introduction

Spectrum analysis, which transforms the signal from time domain to frequency domain, is one of the most important ways to feature extraction in the academic and engineering circles. It provides the foundation for physical phenomenon recognition and engineering decision by extracting the characteristic related to frequency.

Generally speaking, spectrum analysis can be grouped into two classes: parametric methods based on models and non-parametric methods base on Fourier transform. Parametric methods, such as the maximum likelihood method [1,2], the linear predictive frequency estimation techniques [1], AR spectral and the Prony method et al., [3] provide high resolution estimates and are suitable for fewer sampling data. Despite high resolution estimates were obtained, they are rather computationally expensive and require the estimation of the model orders. Non-parametric method employing FFT is computationally efficient and can be easily implemented. Thus the non-parametric method has become the main spectrum analysis method in engineering. But the inherent defect of the FFT called picket fence and leakage will bring the parameters estimation errors. Many methods have been proposed to eliminate the effect of the leakage and picket fence. The most widely accepted and effective methods are the Interpolation Discrete Fourier Transform (IDFT) [4–20]. This kind of methods derived from the approximate solution for windows functions by Rife and Vincent [6]. In 1979, Jain et al. firstly proposed the rectangular window interpolation algorithm [4], which can effective reduce the picket fence and the short range leakage. In order to better suppress the long range leakage caused by harmonic interference, the Hanning window interpolation algorithm [5] was proposed by Grandke et al. A Thought the estimation accuracy was high enough, but it attribute to the Hanning window whose side lobe decays faster than that of rectangular window. Since then, a variety of windowed interpolation algorithm were proposed, such as the Rife–Vincent window [9,12], the Blackman window [13] and minimize side lobe window [11]. Agrez [14] and Belega et al. [9] proposed

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the weighted multipoint interpolation method employs the maximum bin and the other multiple bins on both side of the maximum one. But they are difficult to obtain the analytical solution and need complex calculations, e.g. the Blackman window needs Golden Section Method to search the estimates [13]. Recently, a fast interpolation method, independent of the window type and order, based on suitable lookup tables was proposed by Ferrero et al. [7]. Candan [16,21] suggests a nonlinear relation involving three DFT samples to produce a real valued, fine resolution frequency estimate. A new phase correction method which can reduce the estimation bias was derived by Jan-Ray and Chunming [22]. Beyond that, the iterative frequency estimation by interpolation on Fourier coefficients was also proposed by Aboutanious and Mulgrew [23] and Zakharov and Tozer [24].

In this paper, we analyze the generation principle of the picket fence effect and leakage in discrete Fourier transform and propose an iterative interpolation method which can suppress the leakage effectively. In Section 2, we investigate the causes of picket fence and leakage. In Section 3, we propose the new iterative interpolation method and show the basic theory of it. Finally, in Section 4, a number of simulations are carried out to evaluate the performances of the proposed methods comparing with others method.

2. Picket fence and spectrum leakage in DFT

Discrete Fourier Transform (DFT) is effectively on studying the signal frequency characteristics. It can decompose the signal to various components and estimate the parameters. However, for the finite length of the discrete signal, due to the effect of picket fence and spectral leakage, it is difficult to obtain accurate estimates by DFT. In this section, the principle of picket fence and spectral leakage generated were discussed.

2.1. Picket fence effect

For the sake of brevity, let us consider the discrete Fourier transform of a single signal as

$$x_{\text{sample}}(n) = A_1 e^{j2\pi f_1 n + j\varphi_1}, \quad n = 0, \cdots, N-1 \tag{1}$$

where $x_{\text{sample}}(n)$ is the sampled signal and N is the sampling length. The windowed discrete Fourier transform is

$$X(k) = A_1 e^{j\varphi_1} W(k - \lambda_1)/G, \quad k = 0, \cdots, N-1 \tag{2}$$

where $X(k)$ is the Fourier coefficient; $\lambda_1 = f_1/\Delta f$; Δf is the DFT frequency resolution and $\Delta f = f_s/N$; f_s is the sampling frequency; G is the correction factor of the window function and was defined by

$$G = \sum_{n=0}^{N-1} w(n\Delta t) \tag{3}$$

In general, $X(k)$ consists of a series of spectrum lines with the interval Δf. The discrete frequency for one spectrum line is usually not overlapped with true frequency. This will take observation error on true frequency and is usually called picket fence effect. In Fig. 1, the dashed line

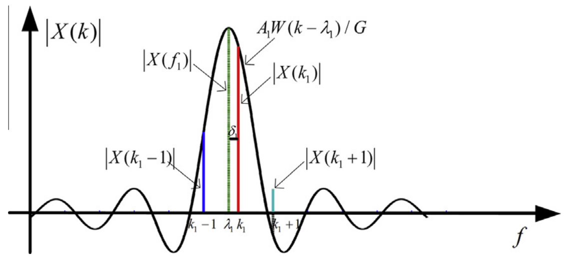

Fig. 1. Observation errors generated by picket fence effect.

is at the real location and the $|X(k_1)|$ is the discrete frequency ($=k_1\Delta f$) calculated by DFT. It is clear that they are not at the same position and there is a bias δ_1.

2.2. Spectral leakage effect

The leakage comes from two ways: one is from the negative frequency of the signal. Another is the harmonic interference in the multi-frequency signals. Let us consider a signal with two components

$$\begin{aligned} x_{two}(n) &= x_1(n) + x_2(n) \\ &= A_1 \sin(2\pi f_1 n + \varphi_1) + A_2 \sin(2\pi f_2 n + \varphi_2) \end{aligned} \tag{4}$$

The windowed discrete Fourier transform is

$$\begin{aligned} X(k) = -\frac{j}{2G}\big[&A_1 W(k-\lambda_1)e^{j\varphi_1} + A_1 W(k+\lambda_1)e^{-j\varphi_1} \\ &+ A_2 W(k-\lambda_2)e^{j\varphi_2} + A_2 W(k+\lambda_2)e^{-j\varphi_2}\big] \end{aligned} \tag{5}$$

where λ_1 and λ_2 are usually not integers. Suppose that the k_1-th spectrum line is most close to λ_1, its discrete Fourier transform is

$$\begin{aligned} X(k_1) = -\frac{j}{2G}\big[&A_1 W(k_1-\lambda_1)e^{j\varphi_1} + A_1 W(k_1+\lambda_1)e^{-j\varphi_1} \\ &+ A_2 W(k_1-\lambda_2)e^{j\varphi_2} + A_2 W(k_1+\lambda_2)e^{-j\varphi_2}\big] \end{aligned} \tag{6}$$

Eq. (6) shows that $X(k_1)$ includes four items: one is the main lobe of the window Fourier transform with the center λ_1, the discrete frequency k_1 is just in it. This is about the picket fence effect discussed above. The second is the main lobe with the center $-\lambda_1$, which is in the negative frequency axis and far away from k_1. The contribution of the second item to $X(k_1)$ was achieved by the side lobes. This is the leakage effect. The last two items are the contribution of the component x_2 and are both far away from k_1. The contributions of them to $X(k_1)$ are also achieved by the side lobes, so they are also the leakage.

Fig. 2 shows the long range leakage produced by the negative frequency. As can be seen from the figures, there is a small value Δk_1 at k_1 generated by the side lobe of negative frequency x_1. It brings the estimation bias on $|X(k_1)|$.

Fig. 3 shows the effect of the short range leakage on $|X(k_1)|$. As before, there is also a small value Δk_1 at which was produced by the side lobe of x_2. Because of this, $|X(k_1)|$ is smaller than the theoretical value, as Δk_1 is negative in Fig. 3.