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On the modeling of a capacitive angular speed measurement sensor

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ABSTRACT

In the present article, a perceptive capacitive sensor for measuring angular speed of a rotating shaft is proposed. The proposed sensor is capable of measuring rotating shaft angular speed, and its changes. The proposed model's sensing part is a suspended clamped–clamped micro-beam, which is parallel with two fixed substrates from the upper and lower sides through the micro-beam's width surface. An electric circuit is used to give out capacitance change as a result of angular speed change, in output voltage. The micro-beam undergoes non-linear electro-static pressure that is induced due to the applied bias DC voltage. The suggested sensor has high sensitivity for a large range of working machines rotating parts angular speed measurement. The governing nonlinear partial differential equation of the transversal motion of the beam is derived and solved by step by step linearization (SSLM) and Galerkin weighted residual methods and the stable region of the sensor is determined. The effects of the applied bias voltage and geometrical properties of the micro-beam on the sensitivity and the range of the measurable angular speed of the sensor are discussed.

1. Introduction

In the recent decade, Technology has been experiencing explosive progresses in micro-electromechanical systems (MEMS) productions. MEMS are built by using recent developments of the integrated circuits (IC) fabrication technology and include movable parts such as beams, plates, membranes, and other mechanical components. As some examples, we can refer to producing accelerometers less than one millimeter on a side, functioning motors that are invisible without the aid of a microscope, gears smaller than a human hair, delivering an injection without stimulating nerve cells and various other tiny elements. Nowadays MEMS capacitive based sensors are widely used in accurate devices. MEMS capacitive sensors tiny dimensions, high sensitivity, long life time and low costs are some of their important characteristics. Use of the integrated circuit technology in the design and production of MEMS devices allow these devices to be batch-manufactured. MEMS capacitive sensors have prime effect in systems control and monitoring. Lots of MEMS capacitive based structures like RF-MEMS switches [1], accelerometers [2], gyroscopes [3], wall shear stress sensor [4], temperature sensor [5], torsional actuators [6,7], and micro-switches are being used in the high tech machines and precise equipment's. Measurement of angular speed has high importance in working machines and control systems in industry [8]. Angular speed sensors are also used in fault detection of Natural roller bearing [9], rotating element bearings with the expected goal to reduce downtime of machines [9], shaft crack detection of nuclear power plant rotating equipment [10], Measurement of high-speed spindle errors in CNC [11], condition

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monitoring of electric motors [12]. Angular speed sensors can be classified into two groups: contact sensing type and noncontact type. Contact type angular speed sensors are mountable on the rotating surface, and are excited by the instrument rotational motion. Some of the contact type sensors are photoelectric sensors, tachometers, optical tachometers, electrical tachometers, photo electric encoders [13], and optical encoders [14]. In the non-contact type sensors, the contact-type sensors defects have been removed. However, the non-contact type sensors requirement of additional equipment, like lasers sources and reflectors make them complicated and expensive. The non-contact type sensors are widely used in industrial measurements [15], such as circular Morie gratings [16], tomography [15], magnetism method, ultrasound, radar, laser, inertial gyros. Also Yamaguchi and Yamasaki [17,18] based on digital image proposed a gaze control active vision system to estimate speed. Mostly, angular speed measurement sensors data processing is based on timer/counter-based methods or ADC-based methods. In the timer/counter based method, an elapsed time (ET) between successive pulses is measured. In ADC-based direct method, angular speed is extracted from the logged data using an efficient signal processing technique. In spite of all achievements in angular speed measurement methods, there are some more challenges, such as lack of multi-purpose hardware to be used with different measurement methods, need for a speed measurement method to make us capable of measuring wide accurate speed, due to the importance of the monitoring time in system control, offering a fast processing measurement method and a measurement instrument that is small and well suited for the dimensions of the measurement place.

Therefore, in this paper a novel method to measure the angular speed of the rotary machines is proposed. The proposed sensor is of the contact-type. The sensor measurement is based on the change of charge of a parallel plate capacitor; and consequently based on the change of the output voltage. The sensor by means of a bias voltage is capable of measuring a wide range of angular speed. Also due to the micro-beam's higher frequency in compared to the usual macroscaled working machines, sensor has high accuracy and very fast response. Occupying of small volume is another positive point of the proposed sensor.

2. Sensor model description and assumptions

Sensor is consisted of a clamped–clamped micro-beam, which is suspended through the two fixed substrates as shown in Fig. 1 and a simple electric circuit [19] as shown in Fig. 2. The micro-beam is with length L, thickness t, width b and gap from the both substrates g_o. Electric circuit gives out the angular speed in sense voltage.

The entire sensor is covered with an insulated package, not to be affected by the environment's physical situations. In order to avoid unbalancing in the very sensitive rotating machines, twin sensor packages can be mounted on the rotating shaft surface in symmetric positions, and the average of the measured angular velocities can be considered as the shaft angular speed. The capacitive sensing can be based on well studied methods such as capacitive divider Fig. 2a or fully differential capacitance sensing method Fig. 2b [19], however due to the capacitive sensing sensitivity to electromagnetic interference (EMI), in the CMOS MEMS sensors fully differential capacitance sensing method is preferred. Of course in the case of differential capacitance sensing method a pair of sensors must be mounted in sensor's package. The fully differential topology significantly improves the interference rejection of the sensor with higher common-mode rejection ratio and power supply rejection ratio.

3. Mathematical modeling

As the first step, in the proposed sensor, the micro-beam is under the applied bias voltages through the both substrates, so the electrostatic pressure is as [20]:

$$P_e(V, w) = \frac{\varepsilon_o b V_1^2}{2(g_o - w)^2} - \frac{\varepsilon_o b V_2^2}{2(g_o + w)^2} \tag{1}$$

where ε_o is the dielectric coefficient of air, V_1, V_2 are respectively applied bias voltages to the upper and lower substrates, that are equal and is the flexural deflection of the micro-beam.

As the sensor mounted surface starts rotating, the micro-beam becomes affected by the rotary inertia pressure of the micro-beam mass. For a micro-beam with special geometrical and material properties, applied rotary inertia will be a function of rotary surface radius R, micro-beam gap g_o, microdeflection w and angular speed of the rotary surface ω as following:

$$P_{rotation}(R, w, \omega) = \rho bh(R + g_o + w)\omega^2 \tag{2}$$

For the case of $R \gg (g_o + w)$, Eq. (2) can be reduced into Eq. (3).

$$P_{rotation}(R, \omega) = \rho btR\omega^2 \tag{3}$$

The non-linear governing equation of the micro-beam based on Euler–Bernoulli beam theory will be as [21]:

$$\widetilde{E}I\frac{\partial^4 w(x,t)}{\partial x^4} - \left[\frac{\widetilde{E}A}{L}\int_0^L \frac{1}{2}\left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx\right]\frac{\partial^2 w(x,t)}{\partial x^2} + (\rho bh)\frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\varepsilon_o b V_{bias}^2}{2(g_o - w)^2} - \frac{\varepsilon_o b V_{bias}^2}{2(g_o + w)^2} + \rho btR\omega^2 \tag{4}$$

where $\widetilde{E}$ is the effective modulus of elasticity, that for a wide micro-beam with thickness h, width $b \geqslant 5h$, is approximated with plate modulus $E/(1 - \upsilon^2)$. The equation of the static deflection can be extracted from Eq. (4) eliminating inertial terms. Using non-dimensional.

The equation of the static deflection can be written as:

$$\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} - \beta\left[\int_0^1 \left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2 d\hat{x}\right]\frac{\partial^2 \hat{w}}{\partial \hat{x}^2} = \frac{\alpha V_{bias}^2}{(1 - \hat{w})^2} - \frac{\alpha V_{bias}^2}{(1 + \hat{w})^2} + \Omega\omega^2 \tag{5}$$

where the new parameters are as:

$$\hat{w} = \frac{w}{g_o}; \quad \hat{x} = \frac{x}{L}$$

$$\alpha = \frac{6\varepsilon_o L^4}{Eg_o^3 t^3}; \quad \beta = \frac{6g_o^2}{t^2}; \quad \Omega = \frac{\rho btRL^4}{E'Ig_o} \tag{6}$$