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A fuzzy state estimator based on uncertain measurements

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ABSTRACT

A fuzzy linear state estimation model is employed, which is based on Tanaka's fuzzy linear regression model, for modeling uncertainty in power system state estimation. The estimation process is based on uncertainty measurements as well as uncertainty parametric. The uncertain measurements and the parameters are expressed as fuzzy numbers with a triangular membership function that has middle and spread value reflected on the estimated states. The proposed fuzzy model is formulated as a linear optimization problem, where the objective is to minimize the sum of the spread of the states, subject to double inequality constraints on each measurement. Linear programming technique is employed to obtain the middle and the symmetric spread for every state variable. The estimated middle corresponds to the value of the estimated state, while the symmetric spreads represent the tightest uncertainty interval around that estimated states. For illustrative purposes, the proposed formulation has been applied to various test systems such as, 4-bus, 6-bus, IEEE 30-bus, IEEE 39-bus, IEEE 57-bus and IEEE 118-bus. Furthermore, an assessment of the time convergence of the proposed method has been carried out to demonstrate the applicability of the proposed estimator as an on-line tool for estimating the uncertainty bounds in power system state estimation.

1. Introduction

Having an accurate picture of the state of a system is an important part of the system operations. While a simple SCADA (Supervisory Control and Data Acquisition) system has the ability to provide the system operators with raw information about the system operation conditions, only a state estimator has the ability of filtering the information to supply a more accurate picture of the status of the system.

The conventional purpose of state estimation is to reduce the effect of measurement errors by utilizing the redundancy available in the measurement system. In particular, the objective is to reduce the variance of the estimates and improve their overall accuracy. The other major objectives of state estimation methods include: detection of gross errors, detection of invalid topological information and detection of model parameter errors.

If the inaccuracy (or error) in the measurements, for a given estimator, is modelled by some random probability distribution function, then the set of feasible estimates can also be modelled by a probability distribution function. These estimators are, therefore, probabilistic in nature. In fact probability theory is generally utilized to handle inaccuracy. Due to the fact that statistics of the measurement errors are difficult to be probabilistically characterized in practice, imprecision in error modelling cannot be equated with randomness, [1], and instead can be associated with fuzziness [2]. Thus, fuzzy theory can satisfactorily be deployed in such circumstances to overcome this limitation and address various uncertainties in the modelling of such statistics. That is particularly due to its ability in handling uncertainties and vagueness associated with the observation errors. Generally, in the context of state estimation, fuzzy estimators are possibilistic in nature. If the observation errors are assumed to be fuzzy due to uncertainty that

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is inherently present in the system, then the estimates are assumed to be a range of possible values. Consequently, in such situations, it is desirable to provide not just a single 'optimal' estimate of each state variable but also an uncertainty range within which we can be assured that the 'true' state variable must lie. This is attainable by utilizing some a fuzzy function to represent the estimates as fuzzy estimates with their associated uncertainty ranges as opposed to crisp estimates (single point only) produced by the conventional estimators [3].

The main theme of this paper is to model the uncertainties associated with the measured quantities in a way that defines an interval (range) with respect to their nominal values. The range is governed by the tolerance, of the measuring instrument (a quantification of accuracy usually provided by the manufacturer) and other factors that are known to have direct effects on network mathematical model being used in the estimation procedure. By implementing the proposed fuzzy linear techniques the confidence interval (or bounds) of the state variables can be computed. Hence, this study presents an estimator based on fuzzy linear regression formulation for estimating the uncertainty interval around the system state variables. This estimator is based on Tanaka's fuzzy linear regression formulation. The uncertainty is expressed in both measurements and network parameters in a unified fuzzy model. The main objective is to minimize the fuzziness in the estimated states. This can be achieved by minimize the sum of spreads of all fuzzy states, subject to double inequality constraints on each measurement to guarantee that the original membership is included in the estimated membership. Linear programming has been employed to obtain the middle and the symmetric spread of every state variable. The estimated middle corresponds to the value of the estimated state, whereas the symmetric spreads in the membership functions of the state variables represents the uncertainty interval around that estimated state. Thus, the primary goal is to minimize the sums of the uncertainties around the states.

2. Uncertainty and state estimation

Schweppe [4] introduced the concepts of uncertainty in the general context of engineering analysis, estimation and optimization. In [4] the concept of unknown-but-bounded errors for modelling uncertainty in estimation problems was introduced. Measurements are assumed to be inexact and have errors that are unknown but fall within a bounded range.

These concepts have been extended and developed recently and have been applied by a number of researchers. Bargiela and Hainsworth [5] introduced bounds on the measurements, with the intention to increase the robustness of estimation. Brdys and Chen [6], developed a technique based on bounded states, and they introduced the term Set Bounded State Estimation (SBSE). Nagar et al. [7] applied concepts from robust control theory and allowed for uncertainty in both the parameters and the measurements. The uncertainty is isolated with the use of a Linear Fractional Transformation (LFT), which enables the preservation of the structure of the uncertainty and allows for a separate manipulation of the nominal and uncertain part. A Linear Matrix Inequalities (LMI) [8] approach is then used to solve the problem to obtain the upper and lower confidence bounds [9].

In power system state estimation, inequality constraints have been applied in optimization to deal with uncertainties. In [10], inequality constraints are employed in a LAV estimator for handling uncertainty in pseudo-measurements, since they are not measured but are known to vary within bounded intervals. An inequality constrained LAV estimator based on penalty functions, was formulated in [11] to estimate states of external systems. A parameter-bounding model derived from bounded noise measurements was used in [12] with a reformulated constrained WLS, to handle unmeasured loads in the system.

Al-Othman and Irving have introduced in [13–15] different methods for estimating the uncertainty interval around the system state variables. One method is based on using a two-step method is proposed for estimating the uncertainty interval around the system state variables. The first step uses weighted least-squares (WLS) as a point estimator to compute the expected values of the state variables. A linear programming formulation is then utilized to find the tightest possible upper and lower bounds on these estimates [13]. The linear formulation was, however, limited to modelling uncertainty only in the measurements which was due to meters inaccuracies, when in fact other elements (inaccuracies of the network mathematical model) can indeed contribute to the uncertainty. As an extension, authors in [14] have introduced another uncertainty analysis method in which the uncertainties are expressed in both measurements and network parameters. The uncertainties in [14] were assumed to be known and bounded. The problem is formulated as a constrained non-linear optimization problem. To find the tightest possible upper and lower bounds of any state variable, the problem is solved by Sequential Quadratic Programming (SQP) techniques. In [15] authors have conducted a comparison study of both methodologies presented in [13,14] in terms of accuracy in estimating the uncertainty interval with various redundancy levels. The study established that both methods provided almost identical bounds estimates. Also, the study showed that based on CPU execution time analysis WLS-LP was found to be faster than the non-linear method.

The main drawback in those formulations was the major computational burden of the process which arises from the need to perform two (LP) or two (SQP), depending on the formulation used, solutions for every uncertainty interval sought. For example, minimizing a particular state variable of interest, subject to all the measurement inequality constraints, provides the lower bound on that state variable. Likewise, maximizing that state variable, again subject to all the measurement inequalities, provides the upper bound for that state. Consequently, for real world large electrical networks that scenario introduces a significant amount of computation and CPU time, which may ultimately question the practicality of those formulations.

The proposed fuzzy linear state estimator (FLSE) has an attractive feature that combats the above drawback. The proposed (FLSE) computes the interval for all states simul-