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Grey bootstrap method of evaluation of uncertainty in dynamic measurement

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Abstract

A method called the grey bootstrap method is proposed to resolve some problems about evaluation of the uncertainty in the process of dynamic measurement by developing four evaluating indicators, i.e. the estimated uncertainty, the estimated interval, the estimated true value and the mean uncertainty. The method can evaluate the uncertainty without any prior information about probability distribution of random variables, separating trends with known and unknown law. Computer simulation and experiment reveal that the fluctuant path of measured data series is perfectly enveloped in the estimated interval, that the law of trends is exactly traced by the estimated true value, that the variation domain of the random variables is availably assessed via the estimated uncertainty, and that statistical characteristic of dynamic measuring process is quantified with the mean uncertainty. Using the grey bootstrap method, the reliability of the estimated results can usually be up to 100% at the given confidence level.

Keywords: Uncertainty; Evaluation; Dynamic measurement; Poor information; Grey system theory; Bootstrap

1. Introduction

The true value of a measurand can hardly be obtained in measurement, due to some interference and can commonly be described with the estimated true value and the uncertainty in measurement, especially in dynamic measurement. At present, the problem is taken into account to mainly assess the uncertainty in some static measurement without a time history, by some new findings.

For example, Dhanish and Mathew [1] discussed the effect of varying CMM (coordinate measuring machines) standard uncertainty on the mean and uncertainty interval width of all studied parameters; Willink and Lira [2] gave a united interpretation of different uncertainty intervals calculated according

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to the procedure of guide to the expression of uncertainty in measurement; based on the given probability distribution and the actual object, Ratel [3] presented some equations to evaluate the uncertainty of the degree of equivalence between laboratories participating in an international comparison of activity measurement; Cordero et al. [4] resolved the problem of evaluation of the uncertainty associated with a single measurement; Kian et al. [5] introduced a new method of providing the confidence intervals that quantify statistical uncertainty as well as the ill-known uncertainty in the estimates; and Rarnan et al. [6] defined minimizing uncertainty. In addition, considering unknown probability distribution and small sample, Xia et al. [7] proposed fuzzy set method of estimating the non-statistical uncertainty in measurement.

It is important to evaluate the uncertainty in dynamic measurement for many practical engineering problems belonging to dynamic processes, i.e. stochastic processes in statistics. Liu [8] proposed a method of evaluation of the dynamic uncertainty. Results of a dynamic measurement can form a data series, namely time series, which can be regarded as a vector. Commonly, in statistics every element in this data series is a random function, which changes with time. At a definite hour, the element in the data series is a random variable. Therefore, the data series are composed of two factors, one is a random variable that belongs to a probability distribution and the other is a trend that belongs to a change law with time.

There are two evaluation methods of uncertainty in measurement, Type A evaluation and Type B evaluation. If both the probability distribution of the random variable and the law of the trend are known, then the uncertainty can be evaluated according to Type A evaluation. When sample is small and probability distribution is unknown, Type B evaluation can be used. However, neither Type A evaluation nor Type B evaluation can be used when probability distribution and mixed probability distribution as well as trends are unknown. This is an important problem about evaluation of uncertainty in dynamic measurement with poor information. Poor information means incomplete information, such as, in system analysis, known probability distribution only with small sample, unknown probability distribution only with several data, and trends without any prior information, etc.

In available findings of data analysis with poor information, the grey system theory [9] and the bootstrap [10] are two of prevailing methods. And these two methods are widely used in many fields of science and technology [11–17].

The grey system theory studies original data series from an uncertain system with poor information. By using the grey model GM(1,1) [9], it can effectively predict the true value of dynamic measurement under the condition of unknown probability distribution, whereas it can not evaluate the uncertainty at the given confidence level. In addition, the true value predicted by the grey model GM(1,1) contains both random variables and trends, thus trends can not be separated from original data series [11,12].

The bootstrap can imitate probability distribution of a system via resampling under the condition of unknown probability distribution with small sample. However, both computer simulation and theoretical analysis indicated that the loss in accuracy of the Monte Carlo approximation of the bootstrap estimate can be infinite, due to the additional uncertainty introduced by finite resampling [18]. Moreover, the estimated interval is less than the actual interval of original data series by using the bootstrap. Hence, errors of the estimated uncertainty are big consequentially.

It is clear that both the grey model GM(1,1) and the bootstrap are not good at effectively resolving the problem on evaluation of dynamic measurement with poor information. For this end, synthetically considering information prediction of the grey model GM(1,1) and probability distribution imitation of the bootstrap resampling, the grey bootstrap method is proposed to evaluate the uncertainty in the process of dynamic measurement with poor information, and computer simulation and experiment are used to make sure of adaptability of the grey bootstrap method proposed in this paper.

2. Model of dynamic uncertainty using grey bootstrap

In a dynamic measuring process, a measurand, suppose an original data series, that can be represented as a vector, outputted by the measurement system can be given by

$$\boldsymbol{X} = \{x(n)\}; \quad n = 1, 2, \ldots, \tag{1}$$

where $x(n)$ is the nth original data; n is the number of current data, i.e. the hour n.

The problem studied in this paper is an estimation of the uncertainty at the hour n by means of m data, which are close and before the hour n