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# Tactile data processing method for the reconstruction of contact force distributions ☆

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ABSTRACT

The main task of robotic skin is recognition of tactile stimuli acting on the surface of a soft elastic layer through the outputs of embedded sensor arrays. The focus of this work is the development of an algorithm for estimating the spatial distribution of contact forces as well as their intensities and directions starting from sensor data. This requires the solution of an inverse problem where only incomplete information (e.g. normal stress on the sensors) is usually available. The proposed method discretizes external forces at the nodes of a grid. The more practical solution is fixing the node number equal to the number of sensors. In this case, the inverse problem of retrieving the force distribution from sensor data is trivially solved if normal forces are applied. For multi-component force distributions, on the other hand, the problem is in principle ill-posed. A solution is achieved through an optimization procedure accounting for the physical features of the problem by the use of the Moore–Penrose pseudo-inverse matrix and of a vector depending on two continuous and adjustable scalar parameters. The algorithm has been tested on simulated single-contact problems (both Hertzian and non-Hertzian normal force distributions) with encouraging results for both accuracy and robustness of the solution.



## 1. Introduction

Tactile sensing is crucial for safe interaction of robots with the daily environment (humans and objects), being a tool for exploration and interpretation. It may provide the robot the necessary feedback to react to external mechanical stimuli [1], as in manipulation and grasping [2].

The amount and quality of information that has to be retrieved by the tactile sensing system depends on the specific task, but essentially consists in detecting distribution, intensity, direction of the contact forces and their time evolution (e.g. [2–6]). Machine Learning (ML) algorithms [7] can be used for the classification of features of the contacting surfaces e.g. roughness, textures, patterns, shapes [8–10] or qualities/modalities of touch [11,12]. ML techniques are especially appealing whenever complex, non-linear mechanisms characterize the underlying phenomenon to be modeled and an explicit formalization of the input–output relationship is difficult to attain. However, empirical induction is used and the input–output function is modeled by a "learning from examples" approach, where the quality of results depends on the quality of training data and also generality is not warranted. Therefore, despite the impressive results in retrieving partial contact information on specific systems [6,13–20], there seems to be room for great improvements when it comes to more generic tasks.

The main problem in force reconstruction is achieving a reasonable compromise between the speed of execution and the accuracy of the results. The more a task requires a fast reaction, as in grasping and holding an object in a stable equilibrium configuration, the less accurate the interpretation of tactile stimuli can be. So it is relatively simple to manage the grasping of a rigid object, where only the resultants of the tactile forces and their centroids need be instantaneously disposed of, but it is enormously complicated to do the same with a deformable object, where the shape of contact areas and the deflections of the surface have to be accounted for. In other instances the recognition aspect prevails over the reactive capacity of the machine.

The problem we address in this work is rather general and may apply to a variety of contexts, hardware settings and tasks, where the tactile sensing system is the scene of complex events, such as multiple contacts in separate regions, possibly even torsion and pinching. The main limitation is that the external surface where the stimuli occur must be relatively flat.

☆ The presented method is under Patent filing.

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The tactile sensing system (e.g. robot skin) we are considering basically consists in one or more arrays of force sensors fixed on a rigid robot hull and coated with an elastomer layer. The tactile stimuli applied on the outer surface of the elastomer are assumed to be received by the sensors as a pressure on their upper side ($T_{33}$ stress component). Any tactile sensing task making use of this kind of device is haunted by the following problem: given the output of the sensor array, derive the force distribution on the outer surface [21]. The problem is fatally an ill-posed one: in fact the set of the data is discrete (the sensor number is finite) and the set of the unknowns is a continuum (a force distribution is a vector field). Special solutions to ill-posed problems of this kind can be searched operationally or numerically by Kalman's Linear Filtering [22] or Tichonov's regularization method [23,24]. These are general methods which optimize solutions in the sense of both minimum square differences and smoothness of results. The latter method has been employed [25–27] to solve a contact problem where an axially symmetric indenter acts at the surface of a soft layer and its effect is sensed by a grid of transducers at the bottom. In this case, however, the analytical solution of the elastic half-space problem is required, but it is available only in special cases (e.g. normal surface forces only, defined indenter shape, axial symmetry, etc.). Also, mathematical methods cannot account for physical constraints, if not expressly introduced.

Given the surface force distribution, the direct problem of finding the stress field at any point in the layer (in particular, on the bottom where the sensors are located) is well known in the theory of elasticity and methods of solutions have been extensively studied [28]. The Green function of this problem for an elastic half-space is known as the Boussinesq equation [29,30].

Therefore, the general solution is a kernel integral over the outer surface area, $\mathbf{b} = \int_A \mathbf{K}(P)\, \mathbf{x}(P) dS$, where $\mathbf{b}$ is the sensor output vector, $\mathbf{x}(P)$ the vector field of the surface forces and the kernel $\mathbf{K}(P)$ accounts for the geometry and physics of the layer ($P$ is any point of the outer surface). The inverse problem requires the inversion of the kernel, which is called deconvolution [6,13,14].

In their application, problems of this kind have to be featured for numerical calculation. If the force distribution is discretized into an array of point forces, a superposition of the Boussinesq equations for the single point forces can be used as a particularly simple and attractive approximation in the case of linear elasticity. Mathematically it reduces to a linear vector equation $\mathbf{b} = \mathbf{Cx}$, where $\mathbf{C}$ is in general a rectangular matrix. The discretization of the deconvolution problem is equivalent to solving this equation for the force vector $\mathbf{x}$. Here we are at the focal point of the present work.

The inverse of the Boussinesq problem has a direct and unique solution only if $\mathbf{C}$ is a square full-rank matrix. This would imply that the number of sensor outputs (rank of vector $\mathbf{b}$) is the same as the number of force components. If we have one output per sensor (typically, the normal stress) and we want to explore 3-component surface forces, the number of points in the force array should be one third of that in the sensor array, to meet the condition. It must be noted, however, that a serious difficulty is to obtain a regular grid of $n/3$ points from a regular grid of n points. For some tasks this fully-constrained solution certainly would be sufficient. Others may require more resolution though, which is what we are targeting with this approach.

A typical ill-posed problem, the object of the present paper, is the one of 3-component point forces applied to the nodes of a pre-determined grid equal in number to the number of sensors. Initial assumptions are made (some can be relaxed in further developments) in that the problem is considered to be static, the skin surface plane, deformations are small and Poisson ratio of the elastomer layer close to 0.5. The proposed method consists in creating a continuous set of solutions to the ill-posed problem by using the Moore–Penrose pseudo inverse matrix [31] and an orthogonal projector acting on a not completely defined vector which, however, complies with the essential physical conditions (e.g. in any contact problem normal forces are compressive and tangential forces are proportional to them by a friction condition). The eligible solution is the one which maximizes an efficiency functional, at best complying with the physical constraints.

The method is developed under the hypothesis of a single simply connected contact area. The extension to the case of multi-contact stimuli will be dealt with in a forthcoming submission.

## 2. Mathematical methods and solutions

### 2.1. Discretization of the inverse problem

Consider an array of point-like sensors at fixed positions ($i$ is a position index) on a rigid surface, initially assumed to be plane with unit normal vector $\mathbf{n}$ (Fig. 1).

In general any sensor has a stress response $\mathbf{n}\cdot\mathbf{T}(i)$ which can be conveyed into an appropriate circuit and retrieved by an electronic device. The surface is coated with an elastomer layer (Poisson ratio $v \sim 0.5$, which means that the volume is approximately conserved during deformation) of thickness $h$. A system of point forces $\mathbf{F}(j)$ is applied at given positions ($j$ is the position index) on the outer surface of the layer. Let $\mathbf{r}(ji)$ be the separation vector of a point on the surface from a sensor location on the substrate. We can put:

$$\mathbf{r}(ji) = \hat{\mathbf{r}}(ji) + \mathbf{n}h \tag{1}$$

where $\hat{\mathbf{r}}(ji)$ is the separation vector between point ($j$) and the projection of point ($i$) ("epicentre") on the outer surface. The stress generated by the surface forces at a given sensor location ($i$) can be evaluated through the Boussinesq equation:

$$\mathbf{T}(i) = \frac{3}{2\pi}\sum_j \frac{\mathbf{F}(j)\cdot\mathbf{r}(ji)}{r^5(ji)}\mathbf{r}(ji)\otimes\mathbf{r}(ji) \tag{2}$$

where the summation is over the number of forces ($j = 1,\ldots,n$), $r = (\mathbf{r}\cdot\mathbf{r})^{1/2}$ and the symbol $\otimes$ represents the tensor product. By using (1) we obtain:

$$\mathbf{n}\cdot\mathbf{T}(i) = \frac{3}{2\pi}\sum_j \frac{\mathbf{F}(j)\cdot\mathbf{r}(ji)h}{\left(\hat{r}^2(ji)+h^2\right)^{5/2}}\mathbf{r}(ji) \tag{3}$$

Eq. (3), as written for the whole system of sensors ($i = 1,\ldots,m$), corresponds to a linear matrix equation of the type:

$$\mathbf{b} = \mathbf{Cx} \tag{4}$$

where, in the general case, $\mathbf{b} = \{\mathbf{n}\cdot\mathbf{T}(i)\}$ is a $3m \times 1$ column vector, $\mathbf{x} = \{\mathbf{F}(j)\}$ is a $3n \times 1$ column vector and

$$\mathbf{C} = \left\{\frac{3}{2\pi}\frac{h\mathbf{r}(ji)\otimes\mathbf{r}(ji)}{\left(\hat{r}^2(ji)+h^2\right)^{5/2}}\right\} \tag{5}$$

is a $3m \times 3n$ matrix. Both $\mathbf{x}$ and $\mathbf{b}$ vectors can be subdivided into three sub-vectors, according to the three spatial components (indexed as 1, 2, 3). A schematic graphical representation of (4) is given in Table 1.

Normally the number of points where forces are applied can be chosen at leisure within a region where the contact can influence the sensors of the array. So, dealing with short-range interactions, there is no reason not to assume $n = m$. In some cases, when the number of sensors disposed of is limited by system constraints, it might be advantageous to interpolate the outputs of sensors through triangulation algorithms thus increasing the number of processed data points. This method permits a better visual