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Multi-objective control for uncertain nonlinear active suspension systems

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ABSTRACT

Performance requirements for vehicle active suspensions include: (a) ride comfort, which means to isolate the body as far as possible from road-induced shocks and vibrations to provide comfort for passengers; (b) road holding, which requires to suppress the hop of the wheels for the uninterrupted contact between wheels and road; and (c) suspension movement limitation, which is restricted by the mechanical structure. In view of such situations, plus the parametric uncertainties, this paper suggests a constrained adaptive backstepping control scheme for active suspensions to achieve the multi-objective control, such that the resulting closed-loop systems can improve ride comfort and at the same time satisfy the performance constraints in the presence of parametric uncertainties. Compared with the classic Quadratic Lyapunov Function (QLF), the barrier Lyapunov function employed in this paper can achieve a less conservatism in controller design. Finally, a design example is shown to illustrate the effectiveness of the proposed control law, where different initial state values are considered in order to verify the proposed approach in detail.

1. Introduction

With the development of automotive industry, the vehicle active suspension system attracts much of the researchers' attention, due to its potential to improve the ride comfort and vehicle maneuverability [1,2]. Using advanced sensors and microprocessors to obtain the information all the time, an active suspension system has the capability to adjust itself continuously for changing road conditions. Generally speaking, performance requirements for vehicle active suspensions include: (a) ride comfort, which means to isolate the body as far as possible from road-induced shocks and vibrations to provide comfort for passengers; (b) road holding, which requires to suppress the hop of the wheels so that there is uninterrupted contact between wheels and road; and (c) suspension movement limitation, which is restricted by the mechanical structure. However, these requirements are often conflicting, and a compromise of the requirements needs to be reached.

In order to manage the trade-off between conflicting requirements, many multi-objective active suspension control approaches are proposed, and a large number of different arrangements have been investigated [3–9]. Among these works, an acceptable approach is to characterize the ride comfort as the main control target and the other performances as time domain constraints, and the control problem is then transformed into a single target control with time domain constraints. For example, in [10] , the problem of constrained H_∞ control for active suspensions is considered, and a state feedback controller is designed to ensure the disturbance attenuation performance of the closed-loop system and meantime, the constraints required in the vehicle suspension control are guaranteed. In [11], a load-dependent controller design approach is presented to solve the problem of multi-objective control for vehicle active suspension systems via state feedback strategy. This approach of designing controllers, whose gain matrix depends on the on-line available information of the body mass, is based on a parameter-dependent Lyapunov function, where the performance requirements are fused in the controller design. Du and Zhang [12] investigate H_∞ control problem for active vehicle suspension systems with actuator time delay, where a delay-dependent memoryless state feedback H_∞ controller is constructed by considering sprung mass acceleration, suspension deflection and tyre deflection as the optimization object with actuator time delay.

However, since the nonlinear aspect of suspension are neglected by most of the above-mentioned approaches, the well-done performance employed by multi-objective control in linear model may not achieve a corresponding good performance in the active suspension systems with nonlinear dynamics. Actually, to form the basis of accurate control, the spring nonlinearity and the piece-wise linear behavior of the damper need to be considered [13]. In addition, because of the change of the number of passengers or the payload,

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vehicle load is easily varied, which will accordingly change the vehicle mass, and this inevitable uncertainty brings considerable difficulties in the process of controller design. Hence, the active suspension will finally behave as an uncertain nonlinear system.

From the perspective of control approaches, most of the existing papers may be conservative partly ascribed to the selection of control Lyapunov function [14,15]. For simplicity, quadratic Lyapunov functions are often proposed as control Lyapunov candidates. Although the controllers based on QLF are sufficient to solve a large variety of control problems, some difficult problems, such as constrained system control, call for more sophisticated forms of Lyapunov functions [16]. Therefore, it is interesting to develop a low conservatism nonlinear control approach with adaptive ability to improve the ride comfort of active suspensions, while considering the other performances as time-domain constraints.

Recently, a significant development for nonlinear constrained system is the Barrier Lyapunov Functions (BLF) approach [17–20]. The key point of this approach is choosing a special Lyapunov candidate to replace the classic quadratic Lyapunov function, and this special Lyapunov candidate should have the property of growing to infinity when the function arguments approach certain limiting values, in order to guarantee the constraints not to be transgressed.

In this paper, the uncertain mathematical model of suspension systems with nonlinear spring and piece-wise linear damper is established firstly, and then a constrained adaptive backstepping control scheme is proposed for active suspensions, such that the resulting closed-loop systems can achieve the performance requirements in the presence of parametric uncertainties. To reduce the conservatism, the barrier Lyapunov functions are employed to guarantee the time-domain constraints within their allowable bounds. Compared with the classic constrained suspension control, the proposed method results in less conservative initial conditions. Finally, a design example is shown to illustrate the effectiveness of the proposed control law, where different initial state values are considered in order to verify the proposed approach in detail.

The rest of the paper is organized as follows. The nonlinear control problem of active suspensions with constraints is formulated in Section 2. The constrained adaptive backstepping controller is synthesized in Section 3, and a special reference trajectory is planned to keep the motions of car body to stabilize in pre-determined time. A design example illustrating the usefulness and advantage of the proposed methodology is given in Section 4, and Section 5 contains the conclusions.

Notation: For a matrix P, P^T denotes its transpose; the notation $P > 0$ ($\geqslant 0$) means that P is real symmetric and positive definite (semi-definite). In this study, symbols $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand for the minimal and maximal eigenvalue, respectively. $\|\cdot\|_\infty$ denotes the ∞-norm, which obeys $\|x\|_\infty = \max(x_j)$, $j = 1,\ldots,n$.

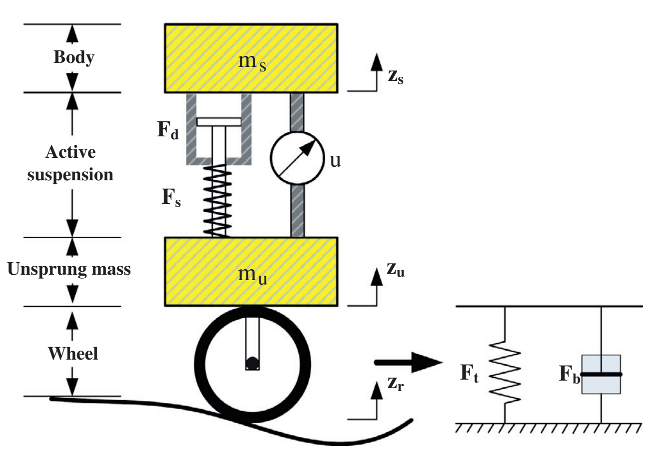

Fig. 1. The model of quarter-car active suspension system.

2. Problem formulation

2.1. Active suspension model

The quarter car model is shown in Fig. 1, which has been used extensively in the literature [11,21]. In this figure, m_s is the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents mass of the wheel assembly; F_d and F_s denote the forces produced by the springs and dampers, respectively, and F_t, F_b are the elasticity force and damping force of the tires. z_s and z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; u is the active input of the suspension system.

The dynamic equations of the sprung and unsprung masses are given as

$$\begin{aligned} &m_s\ddot{z}_s + F_d(\dot{z}_s,\dot{z}_u,t) + F_s(z_s,z_u,t) = u(t),\\ &m_u\ddot{z}_u - F_d(\dot{z}_s,\dot{z}_u,t) - F_s(z_s,z_u,t) + F_t(z_u,z_r,t)\\ &+ F_b(\dot{z}_u,\dot{z}_r,t) = -u(t), \end{aligned} \tag{1}$$

and the forces produced by the nonlinear stiffening spring, the piece-wise linear damper and the tire obey:

$$F_s(z_s,z_u,t) = k_s(z_s - z_u) + k_{sn}(z_s - z_u)^3, \tag{2}$$

$$F_d(\dot{z}_s,\dot{z}_u,t) = \begin{cases} b_e(\dot{z}_s - \dot{z}_u), \\ b_c(\dot{z}_s - \dot{z}_u), \end{cases} \tag{3}$$

$$F_t(z_u,z_r,t) = k_f(z_u - z_r), \tag{4}$$

$$F_b(\dot{z}_u,\dot{z}_r,t) = b_f(\dot{z}_u - \dot{z}_r), \tag{5}$$

where k_s and k_{sn} are the stiffness coefficients of linear and nonlinear terms; b_e and b_c are the damping coefficients for the extension and compression movements, k_f and b_f are the stiffness and damping coefficient of the tire, respectively.

Defining the state variables

$$x_1 = z_s,\ x_2 = \dot{z}_s,\ x_3 = z_u,\ x_4 = \dot{z}_u, \tag{6}$$

the dynamic equations in (1) can be rewritten in the following state-space form:

$$\begin{aligned} \dot{x}_1 &= x_2,\\ \dot{x}_2 &= \theta(-F_d(\dot{z}_s,\dot{z}_u,t) - F_s(z_s,z_u,t) + u),\\ \dot{x}_3 &= x_4,\\ \dot{x}_4 &= \frac{1}{m_u}(F_d(\dot{z}_s,\dot{z}_u,t) + F_s(z_s,z_u,t) - F_t(z_u,z_r,t) - F_b(\dot{z}_u,\dot{z}_r,t) - u), \end{aligned} \tag{7}$$

where $\theta = \frac{1}{m_s}$ is an uncertain parameter. It is to be noted that with a change in the number of passengers or the payload, the vehicle load will easily vary and this will accordingly change the vehicle mass m_s. In literature one can see numerous works that research into uncertain systems, some examples being [22–26].

Assumption 1. The extent of the uncertain parameter is known, i.e.,

$$\theta \in \Omega_\theta = \{\theta : \theta_{\min} \leqslant \theta \leqslant \theta_{\max}\},$$

where $\theta_{\min}$ and $\theta_{\max}$ are the known lower and upper bounds of θ.

2.2. Problem statement

For active suspension systems, the performance requirements to be considered in the controller design include three aspects.

Firstly, in active suspension design, one of the main tasks is to improve ride comfort, which means to design a controller that is