



ELSEVIER

Contents lists available at ScienceDirect

## Optics &amp; Laser Technology

journal homepage: [www.elsevier.com/locate/optlastec](http://www.elsevier.com/locate/optlastec)

# Fractional Fourier transform of Airy-related beams generated from flat-topped Gaussian beams

Zhirong Liu<sup>a,\*</sup>, Xun Wang<sup>a</sup>, Daomu Zhao<sup>b</sup>

<sup>a</sup> Department of Applied Physics, East China Jiaotong University, Nanchang 330013, China

<sup>b</sup> Department of Physics, Zhejiang University, Hangzhou 310027, China

## ARTICLE INFO

## Article history:

Received 16 October 2014

Received in revised form

26 December 2014

Accepted 27 December 2014

Available online 17 January 2015

## Keywords:

Fractional Fourier transform

Laser beam shaping

Propagation

## ABSTRACT

The analytical expression for Airy-related beams generated from flat-topped Gaussian beams propagating through a paraxial *ABCD* optical system is derived and used to investigate its properties in the fractional Fourier transform (FrFT) optical system. The influences of the Airy-related beam order *N* and the fractional order *p* on the evolution of the beam intensity distribution in the FrFT system are examined in detail. Results show that the FrFT optical system provides a convenient way for modulating the beam profile of Airy-related beams by properly choosing optical parameters: lower order Airy-related beams have a longer non-spreading FrFT range, in which the Airy-related beams can maintain their original intensity distribution with no measurable spreading; and it is also found that the Airy-related beam intensity distribution versus the fractional order is symmetrical about  $p = 1$ . Moreover, the Airy-related beam intensity distribution versus the fractional order is periodical, and the period is 2. The results obtained in this work are valuable for Airy-related beam shaping.

© 2015 Elsevier Ltd. All rights reserved.

## 1. Introduction

Since non-spreading (also named non-diffraction or diffraction-free) wave packets were predicted by Berry and Balazs [1], research on this intriguing class of wave packets has recently attracted a lot of interest due to their novel features [2–21]. It is known that an intrinsic characteristic of all these diffraction-free beams is their infinite power, which makes their experimental realization unfeasible. To solve the problem that ideal Airy beams are not square integrable, exponentially decaying terms are introduced to implement finite power Airy beams by extending Berry and Balazs' infinite-energy Airy model by Sivilogou and Christodoulides [3,4]. Even though the finite-energy Airy beams are not exact non-diffracting solutions, these beams exhibit unique features such as weak-diffraction [3,4], self-bending [3–6] and self-healing [7], which are similar to those of the ideal one. These peculiar characteristics have attracted a lot of attention from the scientific community and have projected the finite-energy Airy beams as advantageous optical beams for applications in optical clearing micro-particles [8], curved plasma channel generation [9], optical micro-manipulation [10,11], and other fields.

In general, a finite energy Airy beam can be generated from the fundamental Gaussian beam through an optical Fourier transformation provided that a cubic phase is imposed [4]. Meanwhile, many Airy-related beams have been proposed or generated by making some changes in the methods of generating Airy beams [12–21]. For example, using the partially coherent Gaussian beam as the incident beam, a broadband “white light” Airy beam can be generated, and its decay parameter depends on the spatial coherence of the incident beam [12]; by adding a special apodization mask in the light path, a reduced side-lobe Airy beam can be generated that has an effectively enhanced central lobe, and the side lobe is reduced compared with the common Airy beam [13]; by Airy transforming of flat-topped Gaussian beams, the intensity profile and the propagation characteristics of the Airy-related beams can be modulated through the beam order [19]; generated from a sharply truncated Airy spectrum with uniform amplitude, the sharp spectral cutoff causes the beam to differ from the ideal Airy beam, having an extra oscillating modulation in addition to the desired decay of its fringes [20].

On the other hand, fractional Fourier transform (FrFT), as the generalization of a conventional Fourier transform, was first proposed as a new mathematical tool for solving physics problems by Namias in 1980 [22], and subsequently its potential applications in optics were first explored in 1993 by Mendlovic, Ozaktas and Lohmann [23–25]. Since then FrFT has found wide applications in signal processing, optical image encryption, beam shaping and beam analysis [26–31]. Recently, much work has been done

\* Corresponding author.

E-mail address: [liuzhirong\\_2003@126.com](mailto:liuzhirong_2003@126.com) (Z. Liu).

about their FrFT for various types of beam frequently used in modern optics [32–41]. However, to the best of our knowledge, no results have been reported until now about the propagation properties of the Airy-related beams generated from flat-topped Gaussian beams in the FrFT optical system. In this work, we investigate the propagation properties of the Airy-related beams through the FrFT optical system. The paper is structured as follows: in Section 2, propagation analytical expression for the Airy-related beams generated from flat-topped Gaussian beams through a paraxial optical ABCD system is derived. In Section 3, the evolution of the target beams' intensity distribution in the FrFT system and its dependence influences of several parameters are discussed in detail, and illustrated numerically by using the derived formulas. Finally, the main results obtained are summarized in Section 4.

**2. Fractional Fourier transform of Airy-related beams generated from flat-topped Gaussian beams**

From the optical point of view, three kinds of optical systems for performing the FrFT are proposed [23–25] and shown in Fig. 1, which are the Lohmann I system, the Lohmann II system, and the quadratic graded index (GRIN) medium. Here  $f_s$  is the standard focal length,  $\phi = p\pi/2$  with  $p$  being the fractional order, and  $z$  is the axial distance between the input and output planes along the optical axis in the GRIN medium. According to Matrix Optics, the transfer matrix for Lohmann I optical system can be described by

$$R_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & f_s \tan(\phi/2) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\sin \phi / f_s & 1 \end{pmatrix} \times \begin{pmatrix} 1 & f_s \tan(\phi/2) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \phi & f_s \sin \phi \\ -\frac{\sin \phi}{f_s} & \cos \phi \end{pmatrix}. \tag{1}$$

For Lohmann II optical system, the corresponding transfer matrix can be described by

$$R_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\tan(\phi/2)/f_s & 1 \end{pmatrix} \begin{pmatrix} 1 & f_s \sin \phi \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ -\tan(\phi/2)/f_s & 1 \end{pmatrix} = \begin{pmatrix} \cos \phi & f_s \sin \phi \\ -\frac{\sin \phi}{f_s} & \cos \phi \end{pmatrix}. \tag{2}$$

For the GRIN system, the transfer matrix, with quadratic index variation  $n(r) = n_0(1 - r^2/(2a^2))$ , can be written as [42]

$$R_3 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(z/a) & a \sin(z/a) \\ -\frac{1}{a} \sin(z/a) & \cos(z/a) \end{pmatrix}. \tag{3}$$

where  $a$  denotes the radius of the GRIN medium. Obviously, Eqs. (1)–(3) have the same form when  $f_s = a$ , and  $\phi = z/a$ . Hence, the above mentioned three optical systems have the same transfer matrix and they are equivalent.

In the Cartesian coordinate system, the  $z$ -axis is taken to be the propagation axis. The electric field profile of one-dimensional (1D) Airy-related beams generated from flat-topped Gaussian beams in the source plane  $z = 0$  takes the form as [19]

$$E_1(x_1; z = 0) = 2\sqrt{\pi}A_0 \sum_{n=1}^N \frac{(-1)^{n-1} \sqrt{a_n}}{N} \binom{N}{n} \exp\left(\frac{2a_n^3}{3}\right) \times Ai\left(\frac{x_1}{x_0} + a_n^2\right) \exp\left(\frac{a_n x_1}{x_0}\right). \tag{4}$$

A symmetric two-dimensional (2D) case can be obtained by replacing the coordinate  $x$  with  $y$ , and multiplying the two scalar fields together. In Eq. (4),  $Ai(\cdot)$  indicates the Airy function,  $x_1/x_0$  represents a dimensionless transverse coordinate,  $x_0$  denotes an arbitrary transverse scale,  $a_n = w_0^2/(4nx_0^2)$  represents the modulation parameter so as to ensure containment of the infinite Airy tail,  $A_0$  is a constant related with the beam power,  $N$  denotes the order

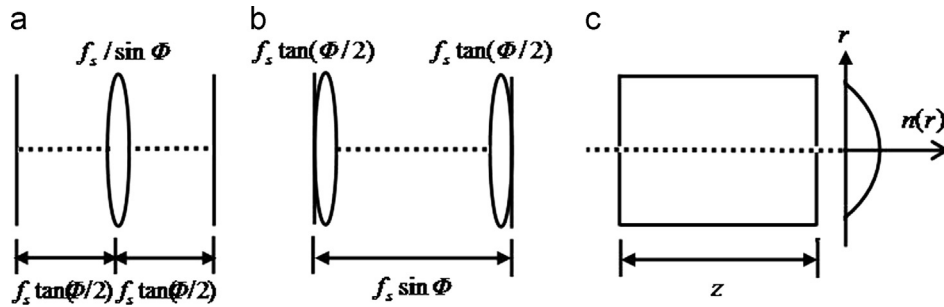


Fig. 1. Three kinds of optical systems for performing the FrFT (a) Lohmann I system, (b) Lohmann II system and (c) GRIN system.

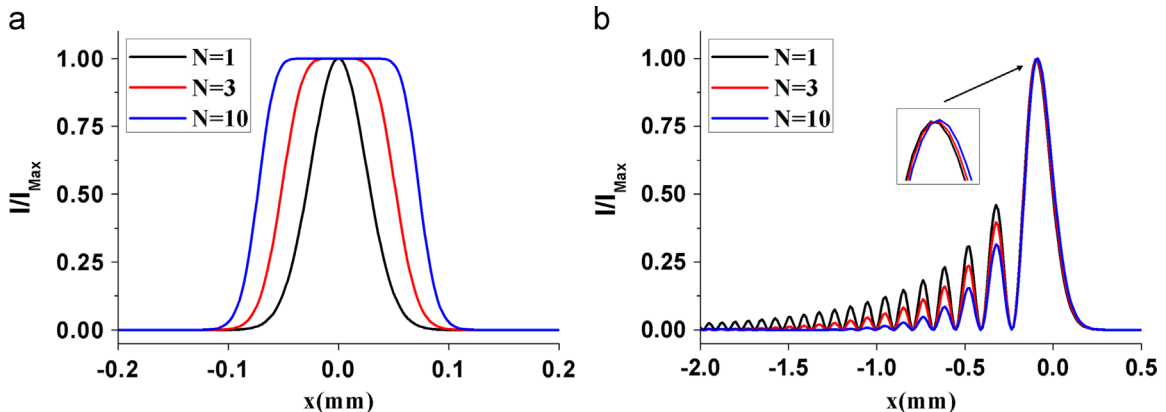


Fig. 2. The normalized intensity distribution of (a) flat-topped Gaussian beams and (b) Airy-related beams generated from flat-topped Gaussian beams of different orders  $N$ .

Download English Version:

<https://daneshyari.com/en/article/732185>

Download Persian Version:

<https://daneshyari.com/article/732185>

[Daneshyari.com](https://daneshyari.com)