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A 2-dimensional optical architecture for solving Hamiltonian path problem based on micro ring resonators

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ABSTRACT

The problem of finding the Hamiltonian path in a graph, or deciding whether a graph has a Hamiltonian path or not, is an NP-complete problem. No exact solution has been found yet, to solve this problem using polynomial amount of time and space. In this paper, we propose a two dimensional (2-D) optical architecture based on optical electronic devices such as micro ring resonators, optical circulators and MEMS based mirror (MEMS-M) to solve the Hamiltonian Path Problem, for undirected graphs in linear time. It uses a heuristic algorithm and employs $n+1$ different wavelengths of a light ray, to check whether a Hamiltonian path exists or not on a graph with n vertices. Then if a Hamiltonian path exists, it reports the path. The device complexity of the proposed architecture is $O(n^2)$.

1. Introduction

The Hamiltonian path in a graph is a path which visits each vertex exactly once. The problem of finding a Hamiltonian path in a given graph is an NP-complete problem, and has many real world applications [1–3]. NP-complete is a complexity class including many real-world problems, which have not been solved with polynomial algorithms on conventional computers, yet. Heuristic and approximation methods, which do not necessarily find exact solution, have been proposed to find an effective solution for the Hamiltonian path problem (HPP) [4–8]. In addition to non-exact methods, several exact methods have also been provided for HPP [9], but no solution with polynomial resource has been found yet.

Using new computational capabilities of optical computing, one can solve NP-complete problems on optical computers by using physical property of light such as high speed, massive parallelism nature, and the ability of splitting a light ray into several rays. Recently some optical approaches have been provided to solve HPP [10–16]. Oltean [12] arranges a graph and the light rays are delayed within the nodes of the graph using a special delaying system through optical fibers. At the destination node, the ray which has visited each node exactly once is searched. In this approach, the length of the optical fibers, used for delaying the signals, increases exponentially while the intensity of the signal decreases exponentially with the number of nodes that are traversed. Another approach is to construct optical masks in the preprocessing phase, and applying the masks consisting of an exponential number of locations, to solve the problem in efficient time [10]. Later, Cohen et al. [14] proposed an optical solver for combinatorial problems such as Hamiltonian cycle based on Nano-technology and lithography methods to produce the masks in Nano-scale size. They solved an instance with 15 vertices. Shaked et al. [16] proposed a method for solving a bounded instance of the Traveling Salesman Problem (TSP) and HPP with maximum size 15. This method exploits fast matrix–vector multiplication based on an optical processor. Recently, one method based on filters [13] is proposed to solve this problem by using light. In this approach first, space solution of the problem is generated and then invalid solutions are eliminated by filters.

In this paper, we propose a novel 2-dimensional (2-D) optical architecture based on optoelectronic devices such as micro ring resonators (MRRs), optical circulators (OCs), and MEMS based Mirrors (MEMS-Ms). Employing this architecture, one can solve the Hamiltonian Path Problem on undirected graphs in linear time. In this architecture, different wavelengths of light are used to display whether a Hamiltonian path does exist or not, and declares a Hamiltonian path, if such a path exists.

The rest of the paper is organized as follows: Section 2 presents the exact definition of the HPP. Sections 3 and 4 discuss and explain the basic element that used in the architecture and the main idea of it, respectively. Section 5 presents simulation results and Section 6 discusses the complexity of the proposed architecture and compares it with other related works. Section 7 explains the conclusion of the paper and suggests future research directions.

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2. The Hamiltonian path problem

Given an undirected graph, $G=(V, E)$, with $|V|=n$ nodes and a start node (v_{start}), the problem is to compute whether there is a sequence of vertices such that from each vertex there is an edge to the next vertex in the sequence, beginning with node v_{start} containing all nodes exactly once. The output for this decision problem is either YES or NO depending on whether the Hamiltonian path does exist or not. A graph may have zero, one or several Hamiltonian paths. For example, in Fig. 1(a), paths v_1–v_4–v_2–v_3, v_1–v_4–v_3–v_2 and v_1–v_3–v_2–v_4 are Hamiltonian paths. Fig. 1 (b) shows a graph which has no Hamiltonian path. Finding a Hamiltonian path is a decision problem in graph theory and the problem of determining whether a Hamiltonian path exists in a given directed/undirected graph is NP-complete [17].

3. The micro ring resonator (MRR)

In this paper, micro ring resonator (MRR) is used as a building block for the proposed solution to the HPP. Fig. 2(a) and (b) shows the schematic structure of MRR and the Z-transform model of an add-drop ring resonator filter as the basic cell, respectively. We investigate MRR as an optical add-drop filter, which consists of a circularly and two straight waveguides. The straight waveguides which serve as input and output pathway for the light ray are patterned close to the micro ring. MRR gets a light ray (including $\lambda_1 \cdots \lambda_i \cdots \lambda_n$ wavelengths) as input from the input port, drops a specific wavelength (i.e., λ_i) from the input ray according to the properties of MRR to its drop port, and sends the rest of the input ray ($\lambda_1 \cdots \lambda_{i-1} \times \lambda_{i+1} \cdots \lambda_n$ wavelengths) to the throughput port.

In order to evaluate throughput and drop port fields, we use the Z-transform [19] to model the basic cell optical performance. The resultant transfer function representing the transmitted field in the throughput port and drop port is simplified to:

$$\frac{E_{t2}}{E_i} = \frac{t - t\gamma z^{-1}}{1 - t^2 \gamma z^{-1}} \tag{1}$$

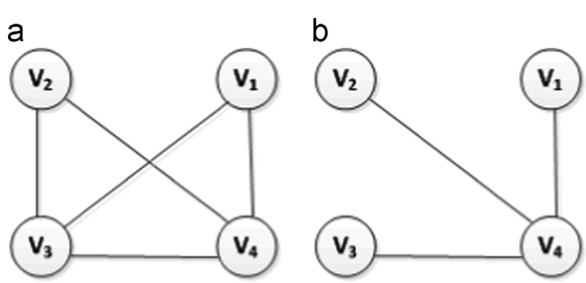

Fig. 1. (a) Graph G that includes some Hamiltonian paths such as v_1–v_4–v_2–v_3. (b) An example of a graph which has no Hamiltonian path.

$$\frac{E_r}{E_i} = \frac{\kappa^2 \sqrt{\gamma z^{-1}}}{1 - t^2 \gamma z^{-1}} \tag{2}$$

so that:

$$\gamma = e^{-\Gamma\alpha}, \quad \alpha = \frac{-2\pi n_{eff} L_R}{\lambda} \tag{3}$$

$$\kappa = \sqrt{1 - t^2}, \quad z = e^{-j\beta L_R} \tag{4}$$

where $L_R=2\pi\ R_{eff}$, t, κ, n_{eff}, λ and β are resonator length, transmission coefficient and coupling coefficient, effective refractive index, wavelength, and phase shift coefficient respectively. γ, Γ and α are imaginary part of refractive index of the lossy medium in the MRR waveguide, optical mode confinement factor in lossy medium and absorption coefficient, respectively. Fig. 3 shows the performance of the basic cell in terms of $R_{eff}=3$ μm, $n_{eff}=3.45$ and $\kappa^2=0.05$ μm (R_{eff} is effective radius of MRR). This figure indicates drop port intensity versus throughput port intensity in range of 1.5–1.56 μm with periodic response characteristics. The drop port response shown by dashed line.

4. The proposed architecture for the Hamiltonian path problem

The main idea of our proposed solution is to consider $n+1$ wavelengths $\lambda_1 \cdots \lambda_{n+1}$, where wavelength λ_i is mapped to vertex v_i ($i=1,\ldots,n$), and wavelength λ_{n+1} is used to representing the solution, if any exists. We construct a lattice using waveguides, similar to the structure of the adjacent matrix of the given graph, and provide an optical structure, DMB (Decision Making Block), on each point corresponding to 1 values in the adjacent matrix. Starting from the up right corner of the lattice where a light ray (containing all wavelengths $\lambda_1 \cdots \lambda_{n+1}$) arrives, DMBs specify the travel direction of the light ray, and MRRs close to row and column waveguides drop the wavelengths in such a way that finally, a light ray reaching the bottom of matrix containing exactly one wavelength λ_{n+1}, represents the solution to the HPP.

In Section 4.1, we design an optical structure DMB, which will be used in the next sections to solve the HPP. In Section 4.2, we provide details of the architecture to solve the yes/no version of the HPP, and in Section 4.3, we extend the architecture to find the order of vertices in the Hamiltonian path, if any exists.

4.1. The DMB structure

We design the optical DMB for propagating and directing the light ray in the proposed architecture. If there is a specific wavelength in the incoming light ray then, the DMB redirects the light ray according to one of directions shown in Fig. 4. In the other case, when a specific wavelength does not exist in the light

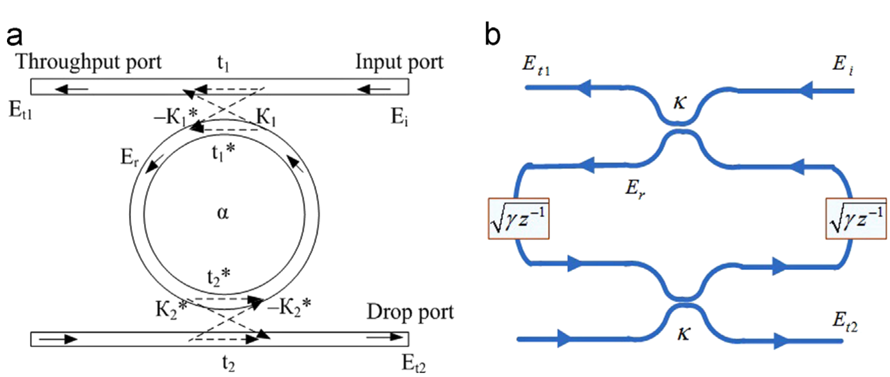

Fig. 2. (a) The schematic structure of MRR [18]. (b) The Z-transform model of the MRR [19].