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Accurate camera calibration with distortion models using sphere images

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ABSTRACT

In order to improve the accuracy of sphere images based camera calibration, a novel approach was proposed in this paper which can calibrate both linear parameters and distortion coefficients simultaneously. The great axis and bitangent lines of the projection conics are applied to solve principal point and sphere center projections. Then the focal length is computed using rotational symmetry of projective cone. Finally distortion coefficients are estimated by optimization search algorithm. Synthetic data experiments analyzed the main error factors and real data results showed that the re-projection error of the proposed method was less than 0.1 pixel which seems to be more accurate than existing methods using spheres and reach the same accuracy level as the planar target method.

1. Introduction

As a fundamental technology in computer vision, camera calibration is to obtain camera model parameters used to recover 3D information from images. Camera model parameters include intrinsic and extrinsic parameters. The intrinsic parameters describe geometry of imaging process, and the extrinsic parameters indicate camera attitude in the world coordinate system. Traditional calibration methods use specific pattern's images captured at several different orientations [1–3]. In these methods, patterns having high precision and being observed in multiple distinct orientations for enough calibration constraints are required, which may have limitation in practical applications. Recently, spheres have been widely used in camera calibration. As sphere is completely symmetric in the space, its silhouette is always visible in any orientation. When camera view is partially blocked, the missing information of image conic can be recovered using fitting algorithm. Moreover, related studies have achieved camera calibration with only one image of several spheres, which seems to be more convenient in applications.

A brief review of related works would be introduced as follows. Penna [4] firstly presented a method for recovering the aspect ratio of the two image axes. Daucher et al. [5] found that the major axis of a sphere image passes through the principal point. Teramoto and Xu [6] derived a formula relating the conic silhouette of a sphere to the image of the absolute conic, and presented a nonlinear method for estimating the camera internal parameters by minimizing the reprojection error. Agrawal and Davis [7] formulated the problem in the dual space and presented a method to recover the camera internal parameters using semi definite programming. Based on the same constraints, Ying and Zha [8–10] developed several linear approaches and presented the geometric interpretations of the giving constraints. Zhang et al. [11] introduced a pole–polar constraint on the imaged absolute conic derived from images conics of two spheres. Wong et al. [12] computed projection of sphere centers using bitangent envelopes of a pair of spheres and presented a formulation of the absolute dual quadric as a cone tangent to a dual sphere with the plane at infinity being its vertex. Duan and Wu [13] proposed a calibration method for para-catadioptric camera using sphere images. Note that existing methods only calibrate linear parameters in the pinhole camera model, and the calibration accuracy is up to $\pm 2\%$ with Zhang's planar target method as a standard.

In order to improve the calibration precision, a novel method is proposed in this paper. Both linear parameters and distortion parameters can be solved using only two sphere images. The principal point and sphere center projections are solved using great axis and bitangent lines of the projection conics based on conclusions of [5,12]. Then focal length and distortion coefficients are determined using rotational symmetry of projective cone.

The remaining of this paper is organized as follows. A brief introduction to camera model and sphere projective model is presented in Section 2. Section 3 details the calibration method. Section 4 provides the results on both synthetic and real data. The conclusions are given in Section 5.

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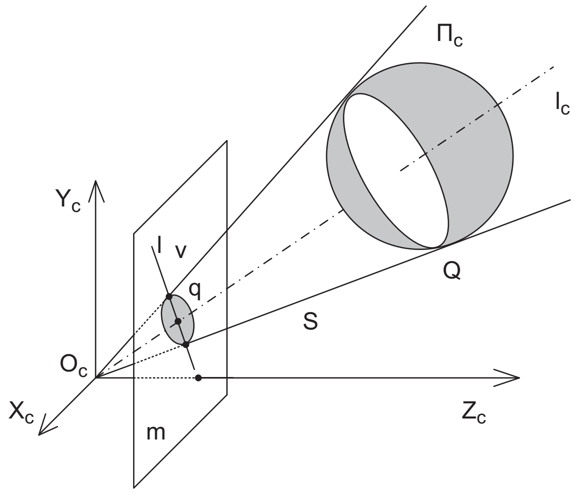

Fig. 1. Projective model of Sphere.

2. Basic principles

Let $\boldsymbol{X}=[X \quad Y \quad Z \quad 1]^T$ be a world point in homogenous coordinates and $\boldsymbol{x}=[u \quad v \quad 1]^T$ be its image point. According to pinhole camera model, they satisfy

$$\mu\boldsymbol{x}=\boldsymbol{PX}, \tag{1}$$

where $\boldsymbol{P}$ is the 3×24 projection matrix, μ is an unknown scale factor. Projection matrix can be decomposed as

$$\boldsymbol{P}=\boldsymbol{K}[\boldsymbol{R}|\boldsymbol{t}], \tag{2}$$

where $[\boldsymbol{R}|\boldsymbol{t}]$ denote the orientation and position of camera in the world coordinate system, and $\boldsymbol{K}$ is the matrix of liner intrinsic parameters, which can be written as

$$\boldsymbol{K}=\begin{bmatrix} f_x & \gamma & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Note that skew factor γ is set to zero in this paper.

Based on pinhole camera model, projection procedure of sphere can be described as a projective cone $\boldsymbol{S}$ with its vertex locating at optical center $\boldsymbol{O_c}$. And the image of the sphere is a conic $\boldsymbol{q}$, as shown in Fig. 1. Let $\boldsymbol{l_c}$ be the symmetric axis of $\boldsymbol{S}$, and $\boldsymbol{l_e}$ be the generator lines of $\boldsymbol{S}$. The intersection point $\boldsymbol{v}$ between $\boldsymbol{l_c}$ and image plane represent the projection of sphere center, and the intersection point between $\boldsymbol{l_e}$ and image plane represent the edge of the image conic. Let plane $\boldsymbol{\Pi_c}$ pass through sphere center and the optical axis of camera. When sphere center is not located on the optical axis, $\boldsymbol{\Pi_c}$ can be uniquely determined. And the intersection line between $\boldsymbol{\Pi_c}$ and image plane represent the great axis of the image conic, which pass through the sphere center projection.

3. Camera calibration

3.1. Principle point

According to [5], the great axis of image conic goes through principle point in image plane. When two image conics are obtained, we have

$$\boldsymbol{x_c}=\boldsymbol{l}_{v1}\boldsymbol{l}_{v2}, \tag{4}$$

where $\boldsymbol{l_{v1}}$, and $\boldsymbol{l_{v2}}$ are great axis of the two conics. When using pixel unit, the ratio of acquisition system digitization steps can be written as

$$s=\frac{d_X}{d_Y}=\frac{f_y}{f_x}, \tag{5}$$

and the coordinate in $\boldsymbol{X}$, $\boldsymbol{Y}$ axis can be unified by

$$\boldsymbol{x}=(u \quad v \quad h)^T \rightarrow \boldsymbol{x'}=(u \quad sv \quad h)^T, \tag{6}$$

where $\boldsymbol{x}$ is the origin coordinate of an image point, $\boldsymbol{x'}$ is the unified coordinate. Similarly, image lines can be unified by

$$\boldsymbol{l}=(a \quad b \quad h)^T \rightarrow \boldsymbol{l'}=(sa \quad b \quad h)^T. \tag{7}$$

In unified coordinate system, the principle point satisfied

$$\boldsymbol{x}_c'=\boldsymbol{l}_{v1}'\boldsymbol{l}_{v2}'. \tag{8}$$

Bring $\boldsymbol{x_c'}$ back to the origin coordinate system, we get

$$\boldsymbol{x}_c'=(u \quad v \quad h)^T \rightarrow \boldsymbol{x}_c=(u \quad v/s \quad h)^T. \tag{9}$$

When more than two image conics are obtained, $\boldsymbol{x}_c'$ can be estimated by minimizing the sum distance between the principal point and each great axis. The quantity can be written as

$$F(\boldsymbol{x}_c')=\sum_{i=1}^{n}\left(\frac{|\boldsymbol{x}_c'\cdot\boldsymbol{l}_{vi}'|}{\sqrt{a_{vi}'^2+b_{vi}'^2}}\right)^2, \tag{10}$$

where $\boldsymbol{l}_{vi}'$ is the great axis of the ith conic in unified coordinate system. After obtaining $\boldsymbol{x}_c'$ with iterative approach, $\boldsymbol{x}_c$ can be solved using (9).

3.2. Sphere center projection

According to [12], the bitangent lines of the projection conics can be applied to solve the projection of sphere center. Let $\boldsymbol{x}_m$ and $\boldsymbol{x}_n$ be the intersections of two inner bitangent lines and two outer bitangent lines separately. Then projection sphere center can be solved using the great axis $\boldsymbol{l}_v$ and line $\boldsymbol{l}_o$ passing through $\boldsymbol{x}_m$ and $\boldsymbol{x}_n$. As shown in Fig. 2, the relationship satisfies

$$\begin{cases} \boldsymbol{x}_{o1}=\boldsymbol{l}_o\boldsymbol{l}_{v1}=((\boldsymbol{l}_{m1}\boldsymbol{l}_{m2})(\boldsymbol{l}_{n1}\boldsymbol{l}_{n2}))\boldsymbol{l}_{v1} \\ \boldsymbol{x}_{o2}=\boldsymbol{l}_o\boldsymbol{l}_{v2}=((\boldsymbol{l}_{m1}\boldsymbol{l}_{m2})(\boldsymbol{l}_{n1}\boldsymbol{l}_{n2}))\boldsymbol{l}_{v2} \end{cases}, \tag{11}$$

where $\boldsymbol{l}_{m1}$, $\boldsymbol{l}_{m2}$ are two inner bitangent lines, $\boldsymbol{l}_{n1}$, $\boldsymbol{l}_{n2}$ are two outer bitangent lines and $\boldsymbol{l}_{v1}$, $\boldsymbol{l}_{v2}$ are great axis of the conics.

3.3. Focal length

The geometry in plane $\boldsymbol{\Pi}_c$ is shown in Fig. 3, where $\boldsymbol{l_a}$, $\boldsymbol{l_b}$ are generator lines of the projective cone, $\boldsymbol{l}_c$ is the symmetric axis and $\boldsymbol{x}_c$ is the principal point. According to the geometry relationship, we have

$$\begin{cases} \tan(\alpha)=\dfrac{|\boldsymbol{x}_a\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|-|\boldsymbol{x}_v\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|}{1+|\boldsymbol{x}_a\boldsymbol{x}_c||\boldsymbol{x}_v\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|^2}=\dfrac{|\boldsymbol{O}_c\boldsymbol{x}_c||\boldsymbol{x}_a\boldsymbol{x}_v|}{|\boldsymbol{O}_c\boldsymbol{x}_c|^2+|\boldsymbol{x}_a\boldsymbol{x}_c||\boldsymbol{x}_v\boldsymbol{x}_c|} \\ \tan(\beta)=\dfrac{|\boldsymbol{x}_v\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|-|\boldsymbol{x}_b\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|}{1+|\boldsymbol{x}_b\boldsymbol{x}_c||\boldsymbol{x}_v\boldsymbol{x}_c|/|\boldsymbol{O}_c\boldsymbol{x}_c|^2}=\dfrac{|\boldsymbol{O}_c\boldsymbol{x}_c||\boldsymbol{x}_b\boldsymbol{x}_v|}{|\boldsymbol{O}_c\boldsymbol{x}_c|^2+|\boldsymbol{x}_b\boldsymbol{x}_c||\boldsymbol{x}_v\boldsymbol{x}_c|} \end{cases}, \tag{12}$$

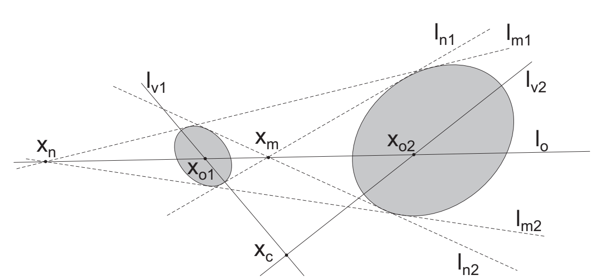

Fig. 2. Projection of sphere center.