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Using the gradient histogram to analyze the continuous phase plate

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ABSTRACT

The geometrical optical method has been used to discuss the far-field distribution characteristics of a continuous phase plate. The gradient histogram of the plate's surface has been calculated. It has been proved that the gradient histogram can be used to show the angular spectrum of a phase plate. The gradient histogram can simplify the analysis process of the angular spectrum and describe the focal spot morphology more intuitively.

1. Introduction

Because of its intuition and efficiency geometrical optical method has been used widely to analyze many optical problems, such as the optical propagation in the graded-index materials [1,2], self-focus [3], parameter analysis of the photonic crystals fiber [4] and the controlling of the adaptive optics [5]. However, there are very little reports on the analysis of continuous phase plate by the geometrical optical method. It is well known that continuous phase plate (CPP) is used in laser system to improve the characteristics of focal spot [6–8]. The chief concern of a CPP is its far-field characteristics. In general, the diffraction integral is used to calculate the intensity distribution in the focal plane. Although this method is efficient, it cannot intuitively reflect the relationship between the surface profile of a CPP and its far-field distribution. But it is just the advantage of the geometrical optical method.

As we know the wave normal can be found form the wavefront phase gradient information [9]. According to this idea the gradient histogram can be used to analyze the far-field of phase plate. Gradient histogram is a typical method based on the geometrical optics. Using this method to study CPP the surface features of this element can be responded. On the other hand, the far-field characteristics of a CPP can be described by the angular spectrum. So the properties, especially the properties with low spatial frequency, of CPP can be analyzed more intuitively and efficiently. In this paper, a 1-dimensional CPP is discussed firstly, and then the analysis method has been expanded into 2-dimensial condition. The physical essence of the gradient histogram has been introduced in detail. With the discussion on its advantages and restrictions, some primary applications of the gradient histogram are presented.

2. Physical model

As shown in Fig. 1, an ideal parallel light passing though a CPP its propagation characteristics will be changed. The incident light can be divided into a number of beamlets. And each beamlet can be shown as a light ray. Refraction will happen when the light ray spread to the rear surface. Using the refraction law the propagation direction of each beamlet can be derived.

Line AO is the incidence light ray and OP is the refracted ray. OB is the normal line, θ and α are the incidence and reflection angle, respectively. δ means the excursion from the original transmission direction. In the incident plane, using small angle approximation, the relationship between those angles can be deduced as

$$\delta = \alpha - \theta = (n-1)\theta \tag{1}$$

θ is also the deflection angle of the surface normal line. In fact it is the surface gradient or the slope at a certain point and it can be described as

$$\theta = dz(x)/dx \tag{2}$$

So the deflection angle of the incident light δ is in direct proportion to the surface gradient θ. Typically, δ is a continuous function.

3. Geometrical optics angle spectrum and gradient histogram

Using a bunch of light rays to express an incident parallel light, after passing through a CPP the deflection angle of each ray depends on the surface gradient at the incident point. We can use δ_1, δ_2 to describe the deflection angle of each ray. As shown in Fig. 1 in x direction, reflection angle is the function of x, it can be written as $\delta(x)$. The light rays with same deflection angle will

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propagate to one point in the far-field. Suppose that every light ray has the same light power, the amount the light rays at one point in the far-field just means the light power at that point. The light power at one point in the far-field can be expressed as $P_{line} \times count(\delta)$, where P_{line} is a constant which means the power of one light ray. $count(\delta)$ means the amount of the light rays which have the same deflection angle δ. The calculating process of $count(\delta)$ is shown in Fig. 2, and it is just as same as the calculation process of the histogram.

Fig. 2(a) gives a example of $\delta(x)$ and it is a discrete sequence $\delta_1, \delta_2 \ldots \delta_{128}$. In order to correspond to Fig. 1 the vertical coordinates of Fig. 2(a) is x, and the horizontal ordinate is δ. Now we divided the angle region that we are interested in into 6 units and count the points in every unit. List the counting results into Fig. 2(b) we get a histogram $hist(\delta)$, which is the angular spectrum of CPP.

As shown in Fig. 2 function $hist(\delta)$ gives the relationship between the relative intensity and the transmission angle. It is the angular spectrum of light intensity. Because the analysis process is based on the geometrical optics principle so we can name it as geometrical optics angular spectrum. It is very similar to the angular spectrum in the Fourier optics. Since the geometrical light ray is tacitly approved as a parallel light, it just corresponds to the parallel condition of Fourier angular spectrum. The difference between the geometrical optics spectrum and the Fourier spectrum is the latter can be used to calculate both the angular spectrum of the intensity and the complex amplitude.

In Fig. 2 we divide the angular region into 6 units discretionarily. In physics, the number of the units reflects the resolution of the angular spectrum. We will discuss the resolution limit in the continuous condition in the rear.

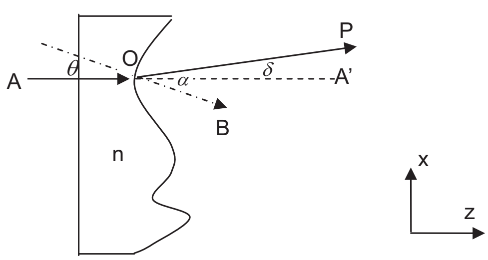

Fig. 1. The light rays in the incidence plane passing through a 1-dimensional CPP.

Using Eq. (1), we can get $hist(\delta) = hist((n-1)\theta)$. In this paper, the relationship can be written as

$$hist(\delta) \sim hist(\theta) \tag{3}$$

which means $hist(\delta)$ and $hist(\theta)$ are similar. According to the calculation process of the histogram, the scaling transformation of the independent variable's coordinates will not change the form of the function, so $hist(\delta)$ and $hist(\theta)$ have the same relative distribution. Therefore, the angular spectrum of a phase plate $hist(\delta)$ can be expressed by the histogram of the surface gradient $hist(\theta)$.

In Fourier optics the angular spectrum formula is $I(\delta) = |F\{\exp[j \times \phi(x)]\}|^2$. Usually the unit of $\phi(x)$ is rad which is often used as the unit of a wavefront or a surface in diffraction optics. In geometrical optics the unit of surface $z(x)$ is different from rad by only a constant with the length dimension. So in form we can define $\theta(x) = d\phi(x)/dx$ and then calculate the histogram, which still shows the relative intensity distribution of the focal spot. Similarly, the increasing of the constant in the expression of focal length $hist((f \times (n-1)\theta)$ will not change the relative distribution and the theoretical analysis of the function. Changing the value of these constants will cause the coordinate scaling or transformation in mathematics and the size variation of the focal spot size in physics. Specially, while the gradient function $\theta(x)$ is multiplied by a constant the size of the focal spot will be changed but the morphology of the spot is unchanged.

Using the histogram to express the intensity distribution in far-field, the energy/power conservation principle must be satisfied. on the condition that shown in Fig. 2 the energy conservation can be expressed as $\sum_{i=1}^{6} hist(\delta) = \sum_{i=1}^{6} hist(\theta) = 128$, which means all the 128 light rays arrived at the far-field.

The above analysis is based on the discrete condition. Now we discuss the histogram $hist(\theta)$ in the continuous condition. The analysis process is shown in Fig. 3.

Fig. 3(a) is a continuous surface in 1-dimensional. Fig. 3(b) is the slop of the surface $\theta = dz(x)/dx$ that shown in Fig. 3(a). One line that we choose arbitrarily goes through the curve and makes a series of intersection points with the serial numbers 1,2,3…. All the intersection points have the same abscissa and the ordinates are $x1$,$x2$,$x3$…. The sum of their neighborhood is the function value of Fig. 3(c). Fig. 3(c) is the histogram $hist(\theta)$.

In the discrete condition, we use $P_{line} \times hist(\theta)$ to describe the light power. In the continuous condition the light power in the

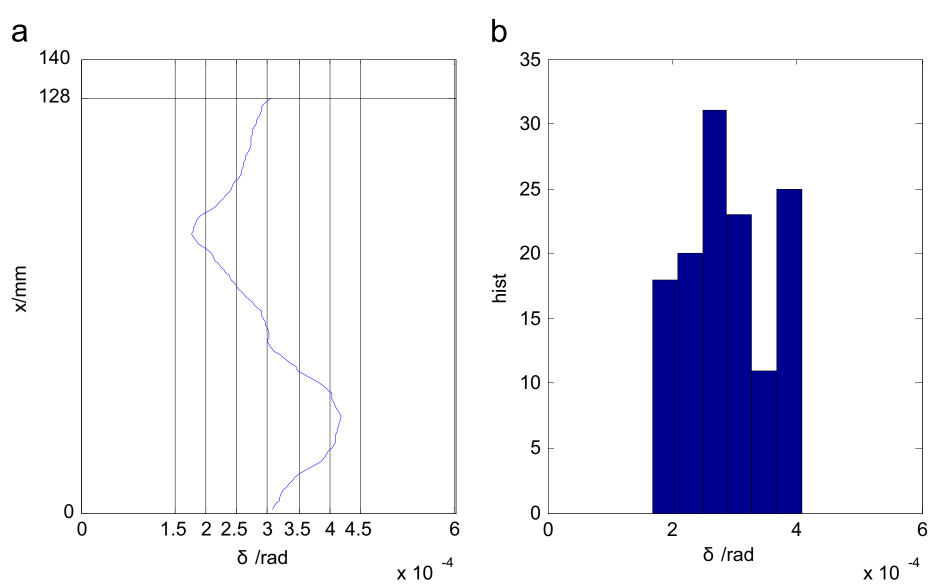

Fig. 2. Calculation of the histogram.