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Directional notch filters for motion control of flexible structures

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ABSTRACT

A new method to design notch filters for MIMO motion control systems with flexible mechanical structures is proposed. The method involves so-called directional notch filters that work only in the direction of the targeted resonant mode. As a result, only one SISO notch filter is required per mode to suppress a resonance throughout the MIMO system. Compared to the conventional approach where a notch filter is placed and tuned in each of the separate control loops, the new approach reduces the order of the controller significantly and facilitates the design process. The directional notch filter is computed using either the input or output mode shapes of the system. A new numerical optimization method to obtain these mode shapes from frequency response data is described. Experiments on a flexible beam setup demonstrate the feasibility of the proposed method in practice.

1. Introduction

Dealing with flexible dynamics of high-precision motion systems has become increasingly important due to increasing performance requirements for these systems. A clear example is given by wafer scanners in the semi-conductor industry. To achieve high accelerations in these scanners, the trend is toward lightweight mechanical designs. However, in general this results in a decrease in stiffness of these systems and causes the resonances of these systems to shift towards lower frequencies, thereby potentially affecting the stability of the system. This complex interaction between the mechanical design, the dynamics of the system, the actuation and sensor systems and the control design poses a challenging problem.

In industrial practice, high-precision multi-input–multi-output (MIMO) motion systems are generally controlled in a decentralized way [1,2]. To this end, MIMO motion systems are decomposed in single-input–single-output (SISO) systems by pre- and post-multiplying the plant with static decoupling matrices. As accurate frequency response data can be obtained at low costs for such systems, loop-shaping is used to design the SISO controllers [3]. The controllers usually contain notch filters to suppress high-frequency resonances that cause instability of the closed-loop system. However, at high frequencies the system is usually not decoupled. The reason for this is the interaction caused by the flexible dynamics of these systems. This means that the notch filters that are designed in each SISO loop separately to suppress the resonances that appear in that loop, do not guarantee that the complete MIMO system is stable. This is because the system is not decoupled at these frequencies.

Ideally, it is possible to decouple all modes and address each of them in a separate control loop by modal decoupling, as described in [4], and applied by, e.g., [5–7]. Modal decoupling exploits the mode shapes to compute static transformation matrices to decouple the modes. However, the number of modes that can be decoupled is limited by the number of actuators and sensors. Flexible structures in principle have infinitely many modes such that perfect decoupling of all modes would require an infinite number of actuators and sensors.

The step toward dynamic decoupling matrices has been made to overcome this limitation, but with limited success only [8,9]. Using observers to estimate the modal states has also been studied extensively. In this field, independent modal space control [10] is the best known example. However, it is well known that these observers suffer from control and observation spillover of unmodelled modes, see [11]. Despite this disadvantage, independent modal space control remains a topic of active research, especially in those cases where a sufficient number of sensors is available, see [12–14] and the references therein.

Another disadvantage of dynamic decoupling and observer based strategies is that they often require a MIMO model of the system to be controlled. Accurate MIMO models for high-precision motion systems are not readily available, see [15,2].

In this paper, an alternative method to deal with the resonances in flexible MIMO motion systems is described. The dynamics of a flexible structure can be described by its modal representation,

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see [16,17]. In this representation, it is assumed that the transfer function of the system can be written as a summation of modes, each with a specific frequency, damping, and mode shape. These mode shapes give the modes a strong directionality in the MIMO system. Since each mode has a specific input and output direction, the control of these modes should take this directionality into account.

Directionality is the main difference between SISO and MIMO systems [18]. While it is often recognized that directions in MIMO systems are important, there are not many methods to design controllers that cope with this directionality, without resorting to full model-based control design methods. [19] describes a method to design a controller that counteracts disturbances in the appropriate directions. Contrary to [19], that is focussed on disturbance rejection, our method aims at using the directions of the flexible modes in the control design.

The directionality of the modes is utilized to design notch filters that have the correct directionality within the MIMO system. For this purpose, a SISO notch filter is distributed over the channels of the MIMO system according to the mode shape of the targeted resonant mode, creating a so-called *directional notch filter*. Conventionally, for each mode one notch filter per SISO loop is required to suppress the resonance in all channels of the system. A directional notch filter requires only one SISO notch filter such that the order of the controller is reduced and the design process is facilitated. Note that contrary to the available feedback, e.g., [20,21] and feedforward methods, e.g., [22] that rely on the repetitive nature of the motion task, our approach is completely independent of the setpoint. The proposed approach aims at enhancement of the dynamics of the system rather than the suppression of (repetitive) disturbances.

In the conventional way of designing notch filters, i.e., independent design of a controller for each SISO loop, merely the stability of the SISO loops is considered. After the loop shaping of the SISO loops, the stability of the MIMO system should be checked in view of the interaction between the loops. If the MIMO stability analysis shows that the system is unstable, it is in general not clear which SISO loop should be adjusted to stabilize the system. Adaptive notch filters as described in [23], are used for the automatic tuning of SISO notch filters. However, the multivariable nature of these systems is not taken into account. Another approach that is often used is sequential loop closing, see [24,25]. Although this technique does consider the interaction between the loops, the order in which the loops are closed is arbitrary, and again it is not obvious which loop should be adjusted in case the performance of the system is unsatisfactory.

Design of the directional notch filters is done in view of MIMO stability, which guarantees the stability of the overall system. In this paper, the characteristic loci [26] are used to design the directional notch filters. Other MIMO stability analysis methods could be used as well.

The transformation matrices are computed from the mode shapes of the modes that need to be suppressed. The mode shapes may be obtained from a parametric model of the system to be controlled, if such a model is available. However, as mentioned before, an accurate parametric model is often not available. Therefore we will reside to a data-based method to obtain the mode shapes. Many methods are known in modal analysis literature, see [27] for an overview. However, these methods require that the number of inputs and outputs exceed the number of modes. Therefore, this paper also describes a new method to obtain the mode shapes from the frequency response data directly.

In addition, experiments have been conducted on a prototype flexible motion system to validate the proposed theory. Summarizing, the main contributions of this paper are:

- a new method to design notch filters for MIMO systems,
- an algorithm to compute mode shapes from frequency response data,
- validation of the method by experiments.

This paper is organized as follows. First, the necessary concepts of the modal description are discussed in Section 2. Next, conventional notch filter design is discussed in Section 3. Section 4 explains the concept of directional notch filtering and elaborates on the computation of directional notch filters. Experiments are presented in Section 5, followed by conclusions in Section 6.

2. Modal description and notation

Flexible structures are characterized by internal deformations of a structure. The dynamics of these internal deformations can be described in *nodal* or *modal* coordinates. Nodal coordinates typically represent the position and velocity of each node in the structure. A large number of nodes is required to obtain an accurate description of the system, which implies that accurate nodal models are generally of high order. To limit the model order, the dynamics of flexible structures are often described using the modal representation, see e.g., [17] or [16]. In this description, the deformation of a flexible structure is described in terms of a limited number of modes and mode shapes. These modes are mutually independent (assuming proportional damping), contrary to the nodal coordinates, which simplifies the analysis. Several modal forms are possible, see [16]. In this paper a modal description with state vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_m \\ \dot{\mathbf{q}}_m \end{bmatrix}, \tag{1}$$

is chosen, where $\mathbf{q}_m$ represent the modal displacements and $\dot{\mathbf{q}}_m$ the modal velocities. Both $\mathbf{q}_m$ and $\dot{\mathbf{q}}_m$ are vectors of length n, where n is the number of modes in the model. The $2n$ equations of motion are given by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}, \end{aligned} \tag{2}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ \mathbf{\Omega}^2 & -2\mathbf{Z}\mathbf{\Omega}^2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_m \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_m & \mathbf{0} \end{bmatrix}, \quad \mathbf{D} = \mathbf{0}, \tag{3}$$

where

$$\mathbf{\Omega} = \mathrm{diag}(\omega_1, \ldots, \omega_n), \tag{4}$$

is the matrix that contains the eigenfrequencies ω_i of the modes and where

$$\mathbf{Z} = \mathrm{diag}(\zeta_1, \ldots, \zeta_n), \tag{5}$$

is the matrix that contains the modal damping ratios ζ_i of the modes. Consequently, the $\mathbf{A}$ matrix is of dimension $2n \times 2n$. $\mathbf{B}$ is $2n \times n_i$, with n_i the number of actuators. Since only force actuators are considered, the first n rows of $\mathbf{B}$ are zero. The lower, non-zero part of $\mathbf{B}$ is denoted with $\mathbf{B}_m$ and is given by

$$\mathbf{B}_m = \begin{bmatrix} \mathbf{b}_{m1}^T \\ \mathbf{b}_{m2}^T \\ \vdots \\ \mathbf{b}_{mn}^T \end{bmatrix}. \tag{6}$$