Mechatronics 24 (2014) 691–700

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

Hard disk drive control by model based dual-rate controller. Computation saving by interlacing

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ARTICLE INFO

Article history:
Received 28 May 2013
Revised 3 December 2013
Accepted 13 December 2013
Available online 22 January 2014

Keywords:
Hard disk drive control
Dual-rate systems
Frequency response
Digital control

ABSTRACT

This paper addresses the application of multirate control techniques to hard disk drives (HDD) single stage actuation. It applies a model-based dual-rate controller with computation saving at its implementation stage. The main goal is twofold: to study a new dual-rate control scheme in this kind of environment that, as it is known, provides the achievement of more storage space and to reach a saving in computation resources when using this specific dual-rate control. The dual-rate control is planed by considering an N times slower measurement -NT- but a faster frequency for control updating -T- being N a positive integer. The work introduces a new discrete lifting modelling step that allows to compute an exact frequency response that helps the controller implementation with computation saving by interlacing. A system output response improvement is obtained by using this methodology. A comparison with an usual single rate control including its implementation by interlacing is done based on a HDD laboratory set-up.

1. Introduction

One interesting problem in the multirate control environment is the hard disk drives control due to the small magnitude of the sampling periods used in this kind of applications. In this context the write/read heads have to respond with accuracy because the high density (number of tracks per inch -TPI-) is growing up year after year. In a single actuator disk file system, the number of sectors and the rotation speed will limit the sampling frequency of the position error signal used for control. In fact, in the sectored servo system for HDD, a circular disk is divided into equally sized angular pieces (called sectors) and position information is written on the disk surface such that the position error signal (PES) is obtained once from each sector. While the number of sectors should be large for increased measurement sampling frequencies, for PES and improved control performance, it should be kept small to reserve an ample space to store data on the disk. This sets a limit to the measurement sampling frequency, which is the product of the rotational speed of the disk and the number of sectors. The actuation input, however, should be good enough to assure a good control performance. So, if it is possible to consider a lower measurement period without getting worse control performance, more storage space will be achieved. This is the reason to consider multirate digital control in this field. Some authors introduced this idea [1,2] and related proper control techniques [3,4] some years ago. More recently new results haven been published. Concretely [5] introduces a ripple free state feedback dual rate control using a dual-rate estimator updated without computing intersample states and consequently reducing the computational burden. A similar method is exposed in [6] where it is used a state feedback control with current observer and it is pointed out the possibility of considering the interlacing technique to overcome the non-uniform nature of computation at every sampling instants with this control scheme. An analysis of computation complexity is added. In [7] three multirate robust controller design methods (mixed $\mathcal{H}_2 = \mathcal{H}_\infty$, mixed $\mathcal{H}_2 = \mu$, and robust $\mathcal{H}_2$ syntheses) for track-following in a HDD read/write head are exposed. In this sense [8] introduces the $\mathcal{H}_2$ controller design for rejecting each disturbance and the ultimate single rate controller is obtained as a control blending problem in a multi-frequency disturbance environment. The sampling rate is chosen such that the frequencies of the mechanical resonances are lower than the Nyquist frequency. In the paper [9] a signficant suppression of the vibrations with frequencies higher than the Nyquist frequency is achieved by a mutirate filter design. This contribution introduces the important topic of frequency response of sampled data systems in this kind of environments.

There are quite a few interesting references about multirate control. Some classical works are for instance [10,11] and many others, but the application of these multirate control techniques to HDD control requires to analyze some specific problems. So, the first problem this contribution deals with is the high sampling

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http://dx.doi.org/10.1016/j.mechatronics.2013.12.003

frequency used in this kind of schemes. This very small sampling period implies that the dicretized transfer function has one zero next to Nyquist frequency. For this reason, it will be necessary to deeply analyze this scenario that usually leads to an undesirable ripple effect, specially in multirate control schemes. Our intention is to use model based controllers [12,13] in this context. One of the problems with multirate control is that the digital processor use is not equally time-distributed. So, the computation saving application [14,15] to a dual-rate controller considering the interlacing technique is a challenging problem with this control option. Actually, this is a new field in multirate control implementation and a new proposal in multirate-control applied to HDD. The different controller interlaced implementations will be studied taking advantage of a new tool for dual-rate frequency computation introduced by the authors [16]. The whole problem requires multi-rate control design techniques, a discrete lifting modelling step, some interlacing controller application criteria and specific multi-rate frequency response techniques in order to give a proper explanation.

The structure of the paper is as follows: Section 2 recalls some preliminary material and sets up definitions and notation in Sections 2.1 and 2.2 presents an explanation of model based dual-rate controllers and its implementation using interlacing of the output of a dual-rate system. This section includes some explanations about the discrete lifting technique in Section 2.3 and with respect to the dual-rate frequency response in Section 2.4. The interlacing technique is also introduced in Section 2.5. Section 3 presents some preliminary results in order to properly understand its application to a laboratory set-up and then this section is dedicated to expose a practical laboratory control. In this case-study, different results could be obtained depending on the interlaced dual rate controller considered. A conclusion section closes the paper.

2. Preliminaries

This section includes the contents the reader needs to properly follow the contribution. The work covers different areas: model-based dual-rate control, dual-rate frequency response, single and dual-rate controller implementation by interlacing techniques. So, in this section, every aspect will be explained.

2.1. Definitions

Consider a discrete-time linear time-invariant (LTI) system, defined by a transfer function $G(z) = B(z)/A(z)$, in the Z-transformed input–output domain, whose poles lie in the interior of the unit circle.

A dual-rate discrete LTI system is one in which the input and the output sequences are assumed to have different sampling periods, T_u and T_y. If they are rationally related, it is possible to define the least common multiple $T_0 = lcm(T_u, T_y)$ usually known as ”metaperiod” or ”frame period” and there exist integers N_u, N_y such that $T_0 = T_u N_u = T_y N_y$ (indeed, then $T_u/T_y = N_y/N_u$ is a rational number). It is usual to define the “greatest common divisor sampling period” $T = gcd(T_u, T_y)$ as well, such that $T_0 = NT$ being $N = lcm(N_u, N_y)$; therefore $T_0 = NT = N_u T_u = N_y T_y$. With these conditions, the behaviour of the dual-rate system may be characterised via a “lifted” transfer function matrix [17,18]:

$$y_l(\tilde{z}) = G_{lifted}(\tilde{z}) u_l(\tilde{z}) \tag{1}$$

where y_l is a vector of length N_y, u_l is a vector of length N_u and G_{lifted} is a $N_y \times N_u$ transfer function matrix. In expression (1) $\tilde{z}$ is related to a sampling period T_0, that is z^N if z is related to T. For convenience, a zero-based array element count will be used in the sequel. For instance, the original sequence $y(k)$ and its lifted one $y_l(k)$ are built in such a way that the ith element of $y_l(k)$, to be denoted as $y_{l,i}(k)$, is $y(k^* N + i), i = 0, \ldots, N-1$.

In multirate field the downsampling (skip) and upsampling (expand) operations are generally used. The Z-transform of sequences $y(kT)$ and $y(kNT)$ is defined as:

$$y^T(z) = \sum_{k=1}^{\infty} y(kT) z^{-k}$$
$$y^{NT}(z_N) = \sum_{k=1}^{\infty} y(kNT) z_N^{-k} \tag{2}$$

where it could be said that $z_N = z^N$, but a correct physical meaning does not allow a trivial substitution. It must be noted that z is related to T and also that $z_N = z^N$ is referred to $NT = T_0$. The expand operator creates a T-sequence from a NT-sequence by inserting zeroes where there is not a T-sequence value.

$$Exp : [y^{NT}(z_N)]^T = \hat{y}^T(z^N) \tag{3}$$

The dual operation is performed by the skip operator that creates a NT sequence from a T-sequence by avoiding $N-1$ T-sequence values between two NT-sequence elements. It is formally defined as:

$$Skp : [y^T(z)]^{NT} = \hat{y}^{NT}(z^N) \tag{4}$$

There are some interesting properties for these operators [19].

In order to recover the original sequences from lifted results in the Z-transformed domain, an expand operator $(z \mapsto z^N)$ [12] may be used. Indeed, as it was said, the operator inserts $N-1$ zeros between the lifted “samples”, so that the original sequence $y(z)$ is obtained from $y_l(z^N)$ with:

$$y(z) = (1 \; z^{-1} \; z^{-2} \; \ldots \; z^{-(N-1)}) y_l(z^N) \tag{5}$$

because of the definition of lifting operation:

$$y_l(z^N) = \begin{bmatrix} y_{l,0}(z^N) \\ y_{l,1}(z^N) \\ \vdots \\ y_{l,N-1}(z^N) \end{bmatrix} = \begin{bmatrix} y_0 + y_N z^{-N} + \ldots \\ y_1 + y_{N+1} z^{-N} + \ldots \\ \vdots \\ y_{N-1} + y_{2N-1} z^{-N} + \ldots \end{bmatrix}$$

2.2. Model-based dual-rate control

Usually the point of departure is an indirect design, which is a continuous controller $G_R(s)$ designed for a continuous process model $G_p(s)$ and the discretized version $G_R(z)$ is calculated considering any feasible discretization method. Sometimes the sampling period makes the discrete loop work with a poor behaviour and even unstable. At these circumstances, if the control action frequency is increased (usually by a factor of N), the dual-rate design will lead to a significant improvement in its behaviour. Other consideration may be to have a fast control loop and to assume an increment (by a factor of N) in the measurement sampling period. In this area, one alternative could be a model-based dual-rate controller. In this case, the design procedure is described in the work developed by [12] according to the scheme showed in Fig. 1 where a non-conventional two-side controller is developed. Basically, some desired closed loop model $M(s)$ is firstly established (the original continuous scheme design), and this loop performance is intended to be developed by means of a dual-rate controller. A slow

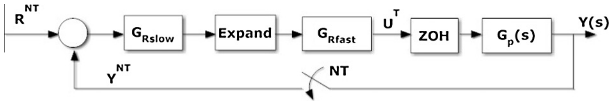

Fig. 1. Dual-rate control scheme.