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Mode switching in causally dynamic hybrid bond graphs

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ABSTRACT

The causally dynamic hybrid bond graph is extended to the case of mode-switching behaviour. Mode-switching 'trees' of switches and elements are historically used by bond graph practitioners to represent elements with piecewise-continuous functions. This case is defined as 'parametric switching' for the purposes of the hybrid bond graph, since the switching is internal to the element, as opposed to 'structural switching' which alters the model structure. This mode-switching 'tree' is concatenated into a new controlled element which features Boolean switching parameters in the constitutive equation, removing unnecessary complexity from the model. Mixed-Boolean state equations can be derived from the model, which are nonlinear and/or time-varying (and hence not in the familiar Linear Time Invariant Form). It can be seen that controlled elements often have a static causality assignment and leave the model structure unchanged. The result is a concise method for representing nonlinear behaviour as a piecewise-continuous function in the bond graph modelling framework.

1. Introduction

This paper[1] is a continuation of the method for construction and analysis of causally dynamic hybrid bond graphs proposed by the authors [2]. The previous paper suggests the terms 'structural discontinuities' and 'parametric discontinuities' for classifying discontinuous behaviour in engineering systems, and established controlled junctions for modelling structural discontinuities. In addition, a dynamic sequential causality assignment procedure (DSCAP) was described, yielding mixed-Boolean state equations. This paper completes the method by looking at parametric discontinuities.

The significant body of work on switched and hybrid bond graphs has already been summarised by the authors [1,2], and references numerous proposals such as the use of petri-nets to select continuous bond graph models [3] and various controlled/switching elements. The authors argue that existing methods are best suited to either qualitative analysis or simulation, but rarely both: the causally dynamic controlled junction offers a method which reflects the physics of the system, allows graphical inspection and can generate mixed-Boolean equations for simulation.

Parametric discontinuities are the case where an element 'switches' between different constitutive equations. This typically occurs in as *mode-switching systems* where an element's behaviour changes so rapidly with time (an order of magnitude faster than the overall time-scale [4]) that it can be considered as an instantaneous transition between continuous modes. The system could be modelled as a purely continuous system and solved using a specialist stiff solver, but this approach still gives slow simulation times and is not feasible for real-time applications such as HiL testing. Mode-switching systems include 'hard nonlinearities,' where there are distinct modes of operation (e.g. stiction/friction). Alternatively, they can occur where some relationship (gained via empirical data or a high-order function) is best described using a piecewise continuous function, such as tyre stiffness.

Just as a structural discontinuity is expected to manifest in the model structure and affect structural properties of the system, a parametric discontinuity is not. As the behaviour of an element changes with time, there is no structural change to the physical system: nothing is connected or disconnected. Therefore, a physical element with discontinuously changing behaviour should be represented by a modelling element with internalised switching.

Mode switching is usually modelled as a collection of continuous modes of operation, controlled by an automaton, petri-net or similar. Within the bond graph framework, mode switching is typically modelled by a 'tree' of ideal switches and standard elements with continuous constitutive equations. Each element gives the equation for a specific mode of operation, and the ideal switches (de)activate it as required. Naturally, only one ideal switch can be ON at any time during a simulation. Soderman [5] and Strömberg [6] formulate mode switching 'trees' of switched

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[1] This paper appears in a preliminary form in the thesis 1. Margetts R. Modelling and Analysis of Hybrid Dynamic Systems using a Bond Graph Approach. University of Bath, 2013.

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0957-4158/

sources, and Mosterman and Biswas [4] present a multi-bond controlled junction selecting a continuous bond graph element from a number of possibilities.

Mode switching has a conceptual advantage in that it aids the development of finite state automata for simulation. However, the 'tree' notation means a model can rapidly grow to a vast size with multiple inputs and outputs for all possible modes of operation. This makes it unsuitable for structural analysis and equation generation purposes. The multi-bond notation suggested by Mosterman and Biswas goes some way to controlling this, but is a little confusing because multibond notation is typically used for multiple degrees of freedom in a model. Their idea is used as a basis for the controlled element defined here.

Hence, a mode-switching tree is used to define a *controlled element* with a mixed-Boolean constitutive equation. This simplifies structural analysis of the bond graph and associated mathematical model, whilst retaining the rigor of the 'tree' notation.

2. The controlled element for parametric discontinuities

This section proposes a new *controlled element* for the modelling of parametric switching. They should not be confused with the existing switched element, which has an on/off behaviour [7].

Consider an element with a piecewise-continuous constitutive function. A mode-switching tree can be constructed using the controlled junctions with associated Boolean terms (as used for structural switching), as shown in Fig. 1. Note that a resistance element is shown, but the principle holds true for inertia and compliance elements.

In this tree, controlled junctions (de)activate the modes of operation, which are given by resistance elements on each branch. These 'branches' are then connected by a regular junction which sums the output values.

- In Fig. 1(a) efforts are summed about a 1-junction: these efforts are the effort exerted by the resistance when a junction is ON plus the zero efforts exerted by the X0-junctions when they are OFF.
- In Fig. 1(b), it is flows which are summed around a zero junction: these flows are the flow exerted by the resistance when a junction is ON plus the zero flows exerted by the X1-junctions when they are OFF.

In a bond graph tree it is important to note that the controlled junctions are constrained so that only one may be ON at any time.

In order to condense the 'tree' into a single controlled element, consider the underlying equations. Quantities are shown on the causal bond graph in Fig. 2. The Boolean parameters associated with the controlled junctions are denoted μ. A reference configuration of $\mu_1 = 1$, $\mu_2 = 0$, $\mu_3 = 0$ is arbitrarily assumed. Note that dynamic causality is internal to the tree: there is static causality on the resistance elements and the input bond.

The Junction Structure Matrices are (for force input and velocity input respectively):

$$\begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \mu_1 \\ 0 & 0 & 0 & \mu_2 \\ 0 & 0 & 0 & \mu_3 \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ F \end{bmatrix},$$

$$\begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \mu_1 \\ 0 & 0 & 0 & \mu_2 \\ 0 & 0 & 0 & \mu_3 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ v \end{bmatrix}$$

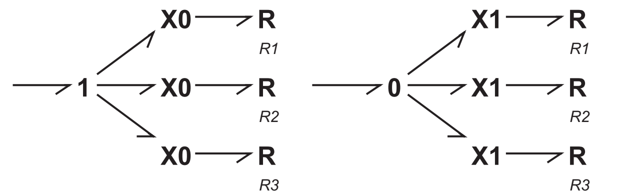

Fig. 1. Bond graph 'Trees' for a piecewise linear resistance element, assuming three modes of operation.

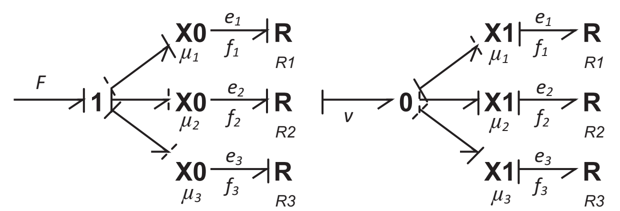

Fig. 2. The piecewise linear resistance element subsystem, showing quantities used in equation generation.

And the Field Laws $\mathbf{D}_{in} = \mathbf{L}\mathbf{D}_{out}$ are:

$$\begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix} = \begin{bmatrix} \mu_1 R_1 & 0 & 0 \\ 0 & \mu_2 R_2 & 0 \\ 0 & 0 & \mu_3 R_3 \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix},$$

$$\begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix} = \begin{bmatrix} \mu_1 R_1^{-1} & 0 & 0 \\ 0 & \mu_2 R_2^{-1} & 0 \\ 0 & 0 & \mu_3 R_3^{-1} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Looking at the summation, we can write:

$$f = f_1 + f_2 + f_3 \qquad e = e_1 + e_2 + e_3$$

$$f = \mu_1 R_1^{-1} e_1 + \mu_2 R_2^{-1} e_2 + \mu_3 R_3^{-1} e_3 \qquad e = \mu_1 R_1 f_1 + \mu_2 R_2 f_2 + \mu_3 R_3 f_3$$

$$\text{And, since flow is constant,} \qquad \text{And, since effort is constant,}$$

$$f = \left(\mu_1 R_1^{-1} + \mu_2 R_2^{-1} + \mu_3 R_3^{-1}\right)F \qquad e = (\mu_1 R_1 + \mu_2 R_2 + \mu_3 R_3)\,v$$

This principle will hold true for 'trees' of compliance and inertia elements. A general definition for the controlled element can therefore be defined as shown in Table 1.

Proposition 1: A Controlled Element for Parametric Switching

A mode-switching tree of controlled junctions and elements can be condensed into a single controlled element. This controlled element has the general constitutive function:

$$output = \sum_{n=1}^{i} \mu_n \Phi_n(input) \qquad (1)$$

Where n is the number of branches to the tree, μ_n is the Boolean term associated with nth controlled junction and Φ_n is the constitutive function of the nth element.