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A simple roll measurement method based on a rectangular-prism

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ABSTRACT

A novel roll measurement method based on a rectangular-prism is presented. A collimated beam is normally incident upon a rectangular-prism with splitting film, and it is divided into two parts, the reflected and the transmitted beams. The displacements of the reflected and the transmitted beams are measured by two quadrant photoelectric detectors QD_1 and QD_2, respectively. The displacement change in the *y*-axis caused by roll can be measured with differential measurements from two quadrant photoelectric detectors. Theoretic analyses show that the crosstalk of straightness errors and yaw errors are avoided, and the crosstalk of the pitch α can be neglected by adopting differential measurements because the direction changes of the reflected and the transmitted beams are both double the changes of pitch. In this way, the stability for the roll measurement can be greatly enhanced. The systemic model is established and the feasibility of such a system is verified by theoretical analysis and experiments. The resolution of 0.3″ and measurement accuracy of 2″ can be obtained by the set-up measurement system.

1. Introduction

Precision moving tables plays an important role in machine tools and measuring machines. In order to achieve higher motion accuracy, it is necessary to measure motion errors of the moving table and to use such errors as a feedback for controlling the motion. Of all the six degree-of-freedom geometric errors (position error, two straightness errors, pitch, yaw and roll), the roll error is the most difficult to measure. Currently, there are some methods available for the roll measurement [1–9]. However, it is still difficult for these measurement methods to be used in practical applications, especially in the simultaneous measurements of six degree-of-freedom geometric errors of a motion table. In this paper, a novel and simple roll measurement method based on laser collimation is presented, which uses a rectangular-prism as the roll measurement sensor. The spatial stability of the laser beam can be ensured by adopting a single-mode fiber coupled semiconductor laser [10], and the anti-interference ability of measurement system is enhanced by the adoption of differential measurements.

2. Measurement principles

2.1. Theoretical analysis

As shown in Fig. 1, a right-handed rectangular Cartesian coordinates system can be established. The collimated laser beam I_1 is normally incident upon the rectangular-prism (RP) along the *z*-axis. The direction vector of beam I_1 can be expressed as $I_1=[0 \quad 0 \quad -1]^T$, which is transposed matrix of $[0\,0\,-1]$. When RP is static, the direction vectors of reflection planes M_1 and M_2 are $N_1=[\sqrt{2}/2 \quad 0 \quad \sqrt{2}/2]^T$ and $N_2=[-\sqrt{2}/2 \quad 0 \quad \sqrt{2}/2]^T$, respectively. The direction vector of the 50% beam splitting film M is $N_0=[0\,0\,1]^T$.

The functionary matrix of the reflection plane can be gotten by

$$M=\begin{bmatrix} 1-2N_x^2 & -2N_xN_y & -2N_xN_z \\ -2N_xN_y & 1-2N_y^2 & -2N_yN_z \\ -2N_xN_z & -2N_yN_z & 1-2N_z^2 \end{bmatrix} \tag{1}$$

So, the beam I_2 can be expressed as

$$I_2=M_1I_1=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \tag{2}$$

In a similar way, we can get

$$I_3=M_2I_2=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \tag{3}$$

$$I_0=M_0I_1=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \tag{4}$$

When the RP moves along the *z*-axis, the three-dimensional translations of RP (Δx, Δy and Δz) have no effect on the normal direction of the planes M_0, M_1 and M_2. Therefore, the reflection beams I_0, I_2 and I_3 have no changes on direction. As shown in Fig. 1,

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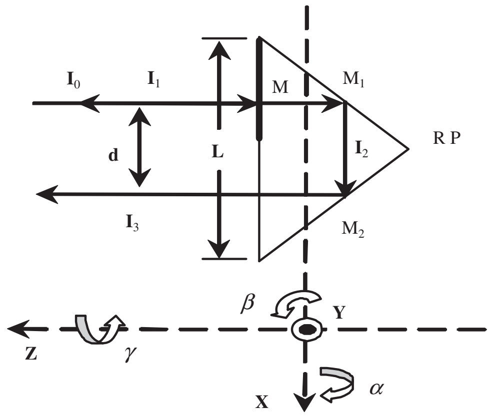

Fig. 1. Diagram of the measuring beam and three-dimensional rotation angles.

assuming that pitch, yaw and roll are α, β and γ, respectively, the corresponding rotation matrixes are as follows:

$$R_\alpha=\begin{bmatrix}1&0&0\\0&\cos\alpha&-\sin\alpha\\0&\sin\alpha&\cos\alpha\end{bmatrix},\quad R_\beta=\begin{bmatrix}\cos\beta&0&\sin\beta\\0&1&0\\-\sin\beta&0&\cos\beta\end{bmatrix},$$
$$R_\gamma=\begin{bmatrix}\cos\gamma&-\sin\gamma&0\\\sin\gamma&\cos\gamma&0\\0&0&1\end{bmatrix}$$

Given the small values of the rotation angles, we can obtain the following result by simplifying:

$$R=R_\alpha R_\beta R_\gamma=\begin{bmatrix}1&-\gamma&\beta\\\gamma&1&-\alpha\\-\beta&\alpha&1\end{bmatrix}\tag{5}$$

If the RP has a movement, the normal direction of the plane M_0 can be expressed as

$$N_0^R=RN_0=\begin{bmatrix}\beta&-\alpha&1\end{bmatrix}^T$$

According to Eq. (1), we can get

$$M_0^R=\begin{bmatrix}1&0&-2\beta\\0&1&2\alpha\\-2\beta&2\alpha&-1\end{bmatrix}\tag{6}$$

$$I'_0=M_0^R I_1=\begin{bmatrix}2\beta&-2\alpha&1\end{bmatrix}^T\tag{7}$$

Similarly, the direction vectors of reflection plane M_1 is $N_1^R=RN_1=(\sqrt{2}/2)\begin{bmatrix}\beta+1&\gamma-\alpha&1-\beta\end{bmatrix}^T$, and we can obtain

$$M_1^R=\begin{bmatrix}-2\beta&\alpha-\gamma&-1\\\alpha-\gamma&1&\alpha-\gamma\\-1&\alpha-\gamma&2\beta\end{bmatrix}\tag{8}$$

After entrance laser beam I_1 enters the RP, the direction vectors of I_1 changes into

$$I'_1=\begin{bmatrix}-(1-1/n)\beta&(1-1/n)\alpha&-1\end{bmatrix}^T\tag{9}$$

The direction vector of the reflected laser beam I_2 can be gotten by

$$I'_2=M_1^R I'_1=\begin{bmatrix}1&\gamma-\alpha+(1-1/n)\alpha&(1-1/n)\beta-2\beta\end{bmatrix}^T\tag{10}$$

Where n is the refractive index of RP and the refractive index of the air is 1.

In a similar way, if the RP has a movement along the z-axis, the direction vectors of reflection plane M_2 is $N_2^R=RN_2=(\sqrt{2}/2)\begin{bmatrix}\beta-1&-\gamma-\alpha&1+\beta\end{bmatrix}^T$, and we can obtain M_2^R by

$$M_2^R=\begin{bmatrix}2\beta&-\alpha-\gamma&1\\-\alpha-\gamma&1&\alpha+\gamma\\1&\alpha+\gamma&-2\beta\end{bmatrix}\tag{11}$$

The direction vectors of the reflected beam is

$$I'_3=M_2^R I'_2=\begin{bmatrix}(1-1/n)\beta&(1-1/n)\alpha-2\alpha&1\end{bmatrix}^T\tag{12}$$

The direction vectors of the exit beam is

$$I_3^R=\begin{bmatrix}0&-2\alpha&1\end{bmatrix}^T\tag{13}$$

Assuming that the hypotenuse length of the rectangular-prism is L and that the distance between incident beam I_1 and exit beam I_3 is d, according to Eqs. (3), (9), (10) and (12), at the moment that exit beam I_3 leaves RP, the displacement of I_3 in the y-axis direction can be expressed as

$$\Delta y=d\cdot\gamma-L\alpha/n\tag{14}$$

As shown in Eq. (14), the displacement Δy contains the information of roll γ and can be measured accurately by the following method. Meanwhile, the influence of pitch α on the roll measurement can be eliminated owing to differential measurement adopted in the following method.

2.2. Measurement principle

As shown in Fig. 2, a collimated laser beam from a single-mode fiber coupled laser diode module is normally incident upon the splitting beam film BS_2 that is coated on the corresponding part of the rectangular-prism. The entrance laser beam is divided into two parts by the BS_2, one part is reflected by the BS_2 and arrives at QD_2 by the BS_1; the other part is transmitted through the RP and arrives at QD_1. So, the positions and their changes for these two parts of the laser beam can be obtained by the two quadrant detectors QD_1 and QD_2, and roll can be measured by following simple calculations.

According to Eqs. (3), (13) and (14), if the initial distance between RP and QD_1 is z_0 and the RP moves a distance of Δz along the z-axis, the displacement on QD_1 of the exit beam I_3 in the y-axis direction can be expressed as

$$\Delta y_1=\Delta y+(z_0+\Delta z)(-2\alpha)=d\gamma+\Delta y_2\tag{15}$$

where

$$\Delta y_2=(-2\alpha)(z_0+\Delta z+L/2n)\tag{16}$$

So, we have

$$\gamma=\frac{(\Delta y_1-\Delta y_2)}{d}\tag{17}$$

According to Eqs. (4) and (7), Δy_2 can be measured by QD_2 whose distance from BS_2 is $z_0+\Delta z+L/2n$. Assuming that

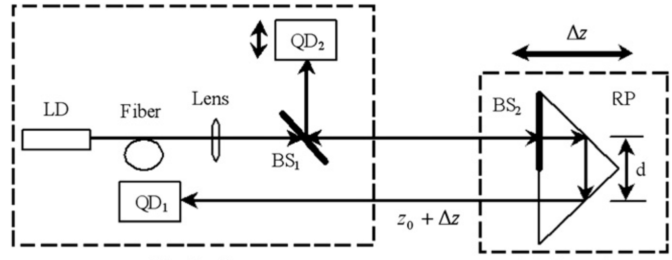

Fig. 2. Diagram of roll measurement.