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Dependence of camera lens induced radial distortion and circle of confusion on object position

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ABSTRACT

Our work presents a theoretical description of camera lens radial distortion and its dependence on the change of the object position with respect to the optical system. We derived approximate formulas within the validity of the third-order aberration theory, which make possible to calculate the changes of lens distortion and the circle of confusion in dependence on the change of the object from the camera lens. These formulas consider a wide angle bundle of rays for calculation of the image point. We also prove that the elimination of distortion by tracing the principal ray does not warrant the removal of distortion of the optical system for the wide angle bundle of rays.

1. Introduction

Various optical measuring instruments are widely used for the measurement of linear dimensions and topography of surfaces, e.g. photogrammetric cameras, linear length sensors, optical interferometers, etc. The most important part of these instruments is the optical system [1,2], which images the measured object on the detector, mostly CCD sensor or film [3]. One can obtain required information about dimensions of the object by a suitable measuring process only on condition that imaging properties of the optical system and parameters of the photodetector are precisely known. Information about imaging properties of the optical system can be obtained by two different ways, either from the manufacturer or by measurements. In practice the first case is almost impossible. Any manufacturer does not probably tell the customer the design parameters of the optical system, i.e. radii of curvature, vertex distances, and indices of refraction of individual elements of the optical system. We have the second chance to measure the parameters. However, we must realize that imaging properties of the optical system depend on the distance of the object from the optical system [4–13]. One needs a high quality camera lens for imaging of the object in measuring systems, which does not induce a geometric deformation of the detected object. The geometric deformation of the image is called distortion [1,2,4–17]. Distortion of the camera lens results from several reasons, which has physical and technological characters. It is not possible to achieve ideal imaging due to physical factors (refraction, diffraction and polarization of light). The image of the point object is not a point but some spot, which is characterized by the specific distribution of energy— the so-called point spread function [4,5,14]. The point spread function depends on parameters of the optical system, e.g. on the shape of the exit pupil, aberrations of the optical system, wavelength, spectral transmittance, material defects, manufacturing errors, and adjusting errors. For our work we will understand the image of the object point as the position of the maximum of the point spread function corresponding to a given object point. The point spread function is characterized by the circle of confusion within the scope of geometric optics and the image of the point object is given by the point, which corresponds to the centroid of the spot diagram [2].

In case of the ideal optical system one can simply determine the image position by the linear transform. Distortion is the difference of the position of the maximum of the point spread function, corresponding to the object point, and the image position using the ideal optical system. If we want to measure the object as accurate as possible, we need to know precisely the imaging properties of the camera lens for the image detection, especially map of distortion in the whole field of view. Due to the fact that it is not possible to manufacture two same camera lenses, it is necessary to test and calibrate the camera lens [18–32], i.e. to determine distortion corresponding to different positions in the image plane of the camera lens. This problem is very important because the measurement accuracy strongly depends on the quality of calibration. It is known that imaging properties of optical systems change in dependence on the object distance from the optical system [4–13]. The result is that distortion also changes with the position of the measured object

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[4–13,17,18]. The previous consequence is an unpleasant property of the measuring process. It means that we should perform the calibration of the camera lens for different distances of the object plane from the camera lens in order to achieve the required measurement accuracy. Distortion and its correction are described deeply in the literature [15–31]. However, almost all works are focused on the calibration of purchased optical systems and use various mathematical models, which are not able to describe the given problem from the physical view, e.g. using geometric optics. Distortion in these works is usually considered in the classical approach using tracing of the principal ray through the optical system.

The aim of this work is to describe the change of the image quality from the view of geometric optics, and the dependence of distortion and the circle of confusion on the distance of the measured object from the camera lens. We want to derive approximate formulas (within the validity of the third-order aberration theory), which enable to calculate the changes of distortion and the circle of confusion due to the change of the measured object from the camera lens. In our work we also prove that the elimination of distortion in the classical approach, i.e. by tracing the principal ray, does not warrant the removal of distortion of the optical system for the wide angle bundle of rays.

2. Theory of distortion of optical system

Firstly we will focus on the description of the theory of distortion in classical geometric optics where distortion means the deviation of the intersection of the principal ray from the point, which corresponds to the image of the ideal rotationally symmetrical optical system [1,2,4–18].

Assume that the object in Fig.1 lies in the plane η and the image is then located in the plane η'. The axial point A is imaged by the optical system as the point A'. Further, we can consider that the optical system is composed of two parts O_1 and O_2 with the aperture stop C between these two parts. The paraxial image of the center of the aperture stop D, which corresponds to imaging by the optical system O_1, is the point P_0. The paraxial image of the point D, which corresponds to imaging by the optical system O_2, is the point P'_0. The point P_0 is the center of the ideal entrance pupil and the point P'_0 is then the center of the ideal exit pupil. The ray from the off-axis point B, which passes through the center of the aperture stop, is called the principle ray. The principal ray intersects the optical axis in points P and P′, which are shifted by the value δp and $\delta p'$ with respect to ideal image points P_0 and P'_0. The angle between the principal ray and the optical axis in the object space is w and in the image space w'. Parameters δp and $\delta p'$determine spherical aberration in pupils of the optical system. Points P and P′ are then centers of effective pupils. It is clear that it exists the position of the effective pupil for every off-axis point B. The principal ray intersects the image plane in the point B', which lies in the distance y' from the optical axis and form the angle w'. The ideal image of the point B is the point B'_0, which is located in the distance $y'_0=my$ from the optical axis, where m is the transverse magnification of the optical system. Distortion is then called the difference

$$\delta y' = y' - y'_0$$

and relative distortion can be defined as

$$\Delta = \frac{\delta y'}{y'_0} = \frac{y'}{y'_0} - 1 \tag{1}$$

As one can see from Fig. 1 it holds that

$$y = -(p - \delta p)\,\mathrm{tg}\,w, \quad y' = -(p' - \delta p')\,\mathrm{tg}\,w', \quad \frac{p'}{p} = \frac{n'}{n} m_P m$$

By substitution of previous formulas into Eq. (1) we obtain the following expression for relative distortion

$$\Delta = \frac{n'}{n} m_P \left(\frac{1 - (\delta p'/f'(m_P - m))}{1 - (n' m_P m \delta p / n f'(m_P - m))} \right) \frac{\mathrm{tg}\,w'}{\mathrm{tg}\,w} - 1 \tag{2}$$

Eq. (2) makes possible to calculate distortion of the optical system in a general case. It is clear that distortion depends on the transverse magnification m of the optical system, i.e. on the object distance, the transverse magnification m_P in pupils and spherical aberration δp and $\delta p'$in pupils of the optical system. If we require distortion to be independent on the transverse magnification, then we obtain the following expression from the necessary condition for the extreme of Eq. (2)

$$\delta p' = \frac{n'}{n} m_P^2 \delta p \tag{3}$$

The previous relation holds on condition that spherical aberration is much smaller than the focal length of the optical system. By the substitution of Eq. (3) into Eq. (2) we can write approximately for distortion

$$\Delta \approx \frac{n'}{n} m_P \left(1 - \frac{n' m_P \delta p}{n f'} \right) \frac{\mathrm{tg}\,w'}{\mathrm{tg}\,w} - 1 \tag{4}$$

It is clear that if the optical system satisfies condition (3), then the optical system has stable correction of distortion (in a classical concept), i.e. distortion is independent on the position of the object with respect to the optical system (independent on the transverse magnification m).

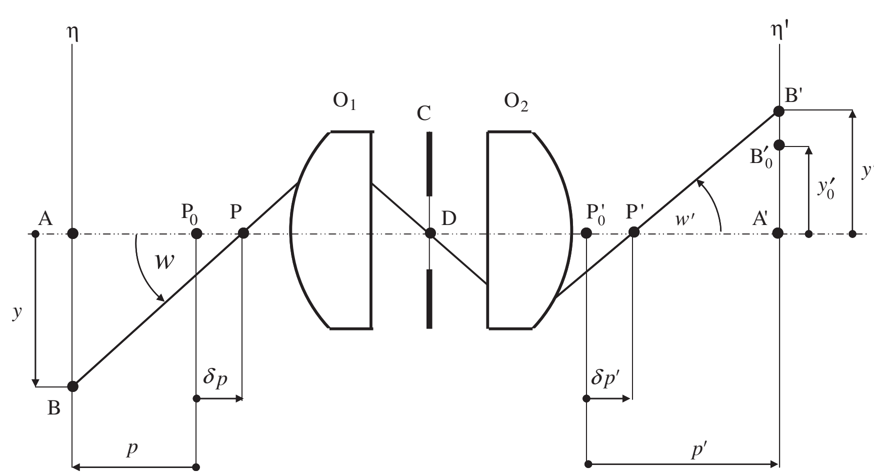

Fig. 1. Schematic for distortion of optical system-geometric theory.