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Degree of polarization for quantum light field propagating through non-Kolmogorov turbulence

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ABSTRACT

Nonclassical polarization properties of a quantum field propagating through non-Kolmogorov turbulence are studied in a turbulent atmosphere paraxial channel. The analytic equation for the quantum degree of polarization of linearly polarized light is obtained. It is shown by numerical simulation that the polarization fluctuations of the quantum field are a function of the turbulent strength, the photon number, the propagation distance, the fractal constant α and the coherence length ρ_0.

1. Introduction

Polarization is a key ingredient of a light beam with a great number of applications both in the quantum and classical realms. The problem of polarization fluctuations in a turbulent atmosphere has attracted considerable attention [1–4]. Hodara [1] derives a theoretical prediction for the turbulence-induced polarization fluctuations wave by geometrical optics. Saleh [2] gave a theoretical and an experimental analysis of optical depolarization due to atmospheric transmission by the geometrical optics approximation and Chernov's three-dimensional ray statistics model. Collett and Alferness [3] characterized the degree of depolarization of the finite collimated Gaussian laser beam in terms of the correlation function. Anufriev et al. [4] estimated the change in the polarization of light arriving from objects located at large distances from the Earth and atmosphere by the scalar Green function. Since James [5] discussion in 1994 on the degree of polarization of a light beam by a partially coherent source generally changes on propagation even in free space, many studies have been done on the degree of polarization of partially coherent beams propagating through turbulence atmosphere [6–8]. Salem et al. [6] studied the degree of polarization changes in partially coherent electromagnetic beams propagating through turbulent atmosphere. Ji and Chen [7] investigated the changes in polarization, coherence and spectrum of the partially coherent electromagnetic Hermite–Gaussian beams propagating through atmospheric turbulence. Kashani et al. [8] studied the effects of turbulent atmosphere on the spectral degree of polarization (SDP) of aberrated partially coherent flat-topped beam. Toyoshima et al. [9] measured the polarization characteristics of an artificial laser source in space-to-ground atmospheric transmission paths. Legre et al. [10] demonstrated a coherent quantum measurement for the determination of the degree of polarization. Hohn [11] investigated the depolarization of a linearly polarized laser beam traversing the atmosphere near the ground path of 4.5 km. Fried and Mevers [12] studied the polarization fluctuation in horizontal propagation of circularly polarized laser light over a 5-mile path.

The purpose of the present paper is to develop a theoretical model for the polarization fluctuations of quantum Gaussian beams in a turbulent atmosphere by quantum Stokes operators.

2. The quantum Stokes operators and the degree of polarization

Polarization properties of photons in free space are conveniently addressed by means of the Hermitian Stokes operators [13]:

$$\begin{aligned}
\hat{S}_0(\boldsymbol{\rho},\mathbf{q}) &= \hat{a}_1^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})+\hat{a}_2^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})\\
\hat{S}_1(\boldsymbol{\rho},\mathbf{q}) &= \hat{a}_1^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})-\hat{a}_2^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})\\
\hat{S}_2(\boldsymbol{\rho},\mathbf{q}) &= \hat{a}_1^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})+\hat{a}_2^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})\\
\hat{S}_3(\boldsymbol{\rho},\mathbf{q}) &= i(\hat{a}_2^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})-\hat{a}_1^\dagger(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q}))
\end{aligned} \tag{1}$$

where $\hat{a}_i(\boldsymbol{\rho},\mathbf{q})$ and $\hat{a}_i^\dagger(\boldsymbol{\rho},\mathbf{q})$ are the photon annihilation and creation operators in the model $(\mathbf{q},i)$, respectively, $\mathbf{q}$ is the momentum of

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photon and $i(i=1,2)$ is its polarization; $\boldsymbol{\rho}$ denotes transverse coordinates of the photon at the z plane. The operators $\hat{a}_i(\hat{a}_i^\dagger)$ obey the well known communication relations $[\hat{a}_i,\hat{a}_k^\dagger]=\hat{\delta}_{ik}$, with $i,k=1,2$. Their mean values $\langle\psi_{ik}|\hat{\mathbf{S}}|\psi_{ik}\rangle$ are the same as the density matrix of the Stokes operators after ensemble averaging, with $\mathbf{S}=\sum_{j=1}^{3}S_j\mathbf{e}_j,\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ are the mutually orthogonal unit vectors along the corresponding axes of coordinates in the polarization space, $\langle\psi_{ik}|\cdots|\psi_{ik}\rangle$ denotes the ensemble average and $|\psi_{ik}\rangle$ is a coherent polarization state of two modes.

The Stokes operators satisfy the commutation relations:

$$[\hat{S}_1,\hat{S}_2]=i2\hat{S}_3,\quad [\hat{S}_2,\hat{S}_3]=i2\hat{S}_1,\quad [\hat{S}_3,\hat{S}_1]=i2\hat{S}_2,\quad [\hat{S}_0,\hat{S}_j]=0\ \ (j=1,2,3) \tag{2}$$

The changes in the polarization of initially polarized light when it propagates through a turbulent atmosphere can be easily obtained [13].

$$P=|\langle\xi|\hat{\mathbf{S}}|\xi\rangle|/\sqrt{\langle\xi|\hat{S}_0(\hat{S}_0+2)|\xi\rangle} \tag{3}$$

where $\hat{S}_0(\hat{S}_0+2)=\sum_{j=1}^{3}\hat{S}_j^2$

$$|\langle\xi|\hat{\mathbf{S}}(\boldsymbol{\rho},\mathbf{q})|\xi\rangle|=\sqrt{\langle\xi|\hat{\mathbf{S}}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle^2+\langle\xi|\hat{\mathbf{S}}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle^2+\langle\xi|\hat{\mathbf{S}}_3(\boldsymbol{\rho},\mathbf{q})|\xi\rangle^2} \tag{4}$$

and

$$\begin{aligned}
|\langle\xi|\hat{\mathbf{S}}_0(\boldsymbol{\rho},\mathbf{q})|\xi\rangle| &= \langle\xi|\hat{a}^\dagger{}_1(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle+\langle\xi|\hat{a}^\dagger{}_2(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle\\
|\langle\xi|\hat{\mathbf{S}}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle| &= \langle\xi|\hat{a}^\dagger{}_1(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle+\langle\xi|\hat{a}^\dagger{}_2(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle\\
|\langle\xi|\hat{\mathbf{S}}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle| &= \langle\xi|\hat{a}^\dagger{}_2(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle+\langle\xi|\hat{a}^\dagger{}_1(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle\\
|\langle\xi|\hat{\mathbf{S}}_3(\boldsymbol{\rho},\mathbf{q})|\xi\rangle| &= i[\langle\xi|\hat{a}^\dagger{}_2(\boldsymbol{\rho},\mathbf{q})\hat{a}_1(\boldsymbol{\rho},\mathbf{q})|\xi\rangle-\langle\xi|\hat{a}^\dagger{}_1(\boldsymbol{\rho},\mathbf{q})\hat{a}_2(\boldsymbol{\rho},\mathbf{q})|\xi\rangle]
\end{aligned} \tag{5}$$

For linearly polarized wave, where $a_2=0$, we have

$$P=|\langle\xi|\hat{S}_0|\xi\rangle|/\sqrt{\langle\xi|\hat{S}_0|\xi\rangle+2} \tag{6}$$

3. Quantum Stokes operators of turbulent atmosphere channel

The propagating quantum field in the turbulent atmosphere paraxial channel is given by [14]

$$\hat{E}_j^\dagger(\boldsymbol{\rho},z)=-\frac{ike^{ikz}}{2\pi z}\sqrt{\tau}\int \hat{E}_j^\dagger(\boldsymbol{\rho}',0)\exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2\right]\exp[\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)]\,d^2\boldsymbol{\rho}' \tag{7}$$

where$\tau=\tau_d\tau_e\tau_a\tau_{ro}\tau_p$is the transmittance of channel, $\boldsymbol{\tau}_e$ is the efficiency transmitting optics, $\boldsymbol{\tau}_d$ is the detector quantum efficiency, $\boldsymbol{\tau}_{ro}$ is the efficiency of receiving transmission, $\boldsymbol{\tau}_a$ is the one-way atmospheric transmission and $\boldsymbol{\tau}_p$ is the attenuation efficiency of point errors, $j=x,y$; $\boldsymbol{\rho}'$ denotes transverse coordinates of the photon at the source plane, $k=2\pi/\lambda$ is the wave number of light, the function $\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)=\chi(\boldsymbol{\rho},\boldsymbol{\rho}',z)+is(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ describes the effects of the atmospheric turbulence on the propagation of a spherical wave, $\chi(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ and $s(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ and terms imposed by atmospheric turbulence and account for the stochastic log amplitude and phase fluctuations, respectively. $\hat{E}_j^\dagger(\boldsymbol{\rho}',0)$ is the field at $z=0$ and is given by the Fourier transform of the annihilation operator for the hypothesis of the passive atmospheric medium:

$$\hat{E}_j^\dagger(\boldsymbol{\rho}',0)=\frac{1}{2\pi}\int d^2\mathbf{q}\hat{a}_j(\mathbf{q},\boldsymbol{\rho}')e^{i\mathbf{q}\cdot\boldsymbol{\rho}'} \tag{8}$$

Here $\hat{a}_j(\mathbf{q},\boldsymbol{\rho}')=\hat{a}_{oj}(\mathbf{q})u(\boldsymbol{\rho}')$ is the effective photon annihilation operator[15,16], $u(\boldsymbol{\rho}')$ is the transverse beam amplitude function for the beam modes and $\hat{n}_{oj}(\mathbf{q})=\hat{a}_{oj}^\dagger(\mathbf{q})\hat{a}_{oj}(\mathbf{q})$ is the initial number operator. $\hat{a}_{oj}(\mathbf{q})$is the photon annihilation operator. A photon eigenstate is a pure state, which is described by wave function $|\psi_{oj}(\boldsymbol{\rho})\rangle$ and $\hat{n}_{oj}(\mathbf{q})|\psi_{oj}(\boldsymbol{\rho},\mathbf{q})\rangle=n_{oj}(\mathbf{q})$.

By Eqs. (5) and (6), we have

$$\begin{aligned}
\hat{E}_j^\dagger(\boldsymbol{\rho},z) &= -\frac{ike^{ikz}}{4\pi^2 z}\sqrt{\tau}\int d^2\mathbf{q}\int d^2\boldsymbol{\rho}'\hat{a}_j(\mathbf{q})e^{i\mathbf{q}\cdot\boldsymbol{\rho}'}\exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2\right]\exp[\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)]\\
&= \frac{1}{2\pi}\int d^2\mathbf{q}e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\hat{a}_j(\mathbf{q},\boldsymbol{\rho},z)
\end{aligned} \tag{9}$$

where

$$\hat{a}_j(\mathbf{q},\boldsymbol{\rho},z)=-\frac{ike^{ikz}}{2\pi z}\sqrt{\tau}\int d^2\boldsymbol{\rho}'\hat{a}_j(\mathbf{q})e^{i\mathbf{q}\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})}\exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2\right]\exp[\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)] \tag{10}$$

The photon creation operator $\hat{a}^\dagger(\mathbf{q},\boldsymbol{\rho},z)$ is expressed as

$$\hat{a}_j^\dagger(\mathbf{q},\boldsymbol{\rho},z)=\frac{ike^{-ikz}}{2\pi z}\sqrt{\tau}\int d^2\boldsymbol{\rho}'\hat{a}_j^\dagger(\mathbf{q})e^{-i\mathbf{q}\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})}\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2\right]\exp[\psi^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)] \tag{11}$$

By Eqs. (5) and (6), we can obtain the number operator in z plane of a beam propagating through atmospheric turbulence

$$\begin{aligned}
\langle\hat{a}_j^\dagger(\mathbf{q},\boldsymbol{\rho},z)\hat{a}_l(\mathbf{q},\boldsymbol{\rho},z)\rangle_m &= \hat{n}_{jl}\left(\frac{k}{2\pi z}\right)^2\int\int d^2\boldsymbol{\rho}'d^2\boldsymbol{\rho}''u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')e^{i\mathbf{q}\cdot(\boldsymbol{\rho}''-\boldsymbol{\rho}')}\\
&\times\exp\left[ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right]\langle\exp[\psi^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)\\
&+[\psi(\boldsymbol{\rho},\boldsymbol{\rho}'',z)]]\rangle_m
\end{aligned} \tag{12}$$

where $\hat{n}_{jl}=\tau\hat{n}_{0jl},\langle\ldots\rangle_m$ denotes averaging over the ensemble of the source of the turbulent atmosphere.

Taking into account the quadratic approximation for Rytov's phase structure function [17,18], we have

$$\langle\exp[\psi^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)+[\psi(\boldsymbol{\rho},\boldsymbol{\rho}'',z)]]\rangle_m=\exp[-(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2/\rho_0^2] \tag{13}$$

For non-Kolmogorov atmospheric turbulence$\rho_0^2$is given by [19]

$$\rho_0=\left\{\frac{\pi^2\kappa^2 zA(\alpha)}{6(\alpha-2)}C_n^2\left[\kappa_m^{2-\alpha}\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)(2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}\right]\right\}^{-1/2}$$

where $\kappa_0=2\pi/L_0$, L_0 being the out scale of turbulence, $\kappa_m=c(\alpha)/l_0$, l_0 being the inner scale of turbulence, $c(\alpha)=\left[\Gamma\left(5-\frac{\alpha}{2}\right)A(\alpha)\frac{2}{3}\pi\right]^{1/\alpha-5}$ and $A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right)$, with $\Gamma(x)$ being the Gamma function of the term, C_n^2 is a generalized refractive-index structure parameter with unit$m^{3-\alpha}$.

The turbulent ensemble average number operator is given by

$$\begin{aligned}
\langle\hat{a}_j^\dagger(\mathbf{q},\boldsymbol{\rho},z)\hat{a}_l(\mathbf{q},\boldsymbol{\rho},z)\rangle_m &= \hat{n}_{ojl}\left(\frac{k}{2\pi z}\right)^2\int\int d^2\boldsymbol{\rho}'d^2\boldsymbol{\rho}''u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')\\
&\times\exp\left[-\frac{(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2}{\rho_0^2}\right]e^{i\mathbf{q}\cdot(\boldsymbol{\rho}''-\boldsymbol{\rho}')}\exp\left[ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right]
\end{aligned} \tag{14}$$

For the fundamental-model Gaussian collimated beam, the field $u(\boldsymbol{\rho}')$ at the plane $z=0$ can be represented as [16]

$$u(\boldsymbol{\rho}',0)=\frac{1}{w_0}\sqrt{\frac{2}{\pi}}\exp\left[-\frac{\boldsymbol{\rho}'^2}{w_0^2}\right] \tag{15}$$

where w_0 is the waist radius of the Gaussian beam.

By Eqs. (14) and (15), we have

$$\langle\hat{a}_j^\dagger(\mathbf{q},\boldsymbol{\rho},z)\hat{a}_l(\mathbf{q},\boldsymbol{\rho},z)\rangle_m=\frac{2\hat{n}_{jl}}{\pi w_0^2}\left(\frac{k}{2\pi z}\right)^2\int\int d^2\boldsymbol{\rho}'d^2\boldsymbol{\rho}''\exp\left[-\frac{\boldsymbol{\rho}'^2+\boldsymbol{\rho}''^2}{w_0^2}\right]$$