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Propagation properties of the beam generated by Gaussian mirror resonator passing through a paraxial *ABCD* optical system

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ABSTRACT

By expanding a hard aperture function into a finite sum of complex Gaussian functions, approximate propagation formula is derived in the situation that the beam generated by Gaussian mirror resonator passes through a paraxial *ABCD* optical system with an annular aperture. The corresponding forms for a circular aperture and a circular black screen are also given. Some numerical simulations are shown to illustrate propagation properties and focusing properties of the beam passing through a paraxial *ABCD* optical system with the three different kinds of aperture.

1. Introduction

An unstable resonator containing a mirror with variable reflectivity is widely used in high power lasers. An unstable resonator with a mirror that has a reflectivity with a Gaussian profile offer a number of useful properties [1–5]. It may be used to generate a flat-topped gaussian beam [6,7]. The gaussian mirror resonator is increasingly used in gas and solid-state lasers. Thus it is necessary to study the laser beam generated by this resonator. The beam generated by Gaussian mirror resonator can be decomposed into a linear combination of the lowest-order Gaussian beams [7]. The propagation properties of the beam were fully discussed in [8,9], but the propagation of the beam through a paraxial *ABCD* optical system with an annular aperture has still not yet been studied in detail.

Based on the scalar diffraction theory, the propagation properties are studied when the beam generated by Gaussian mirror resonator passes through a paraxial *ABCD* system with an annular aperture. To simplify the calculation model, we expand the hard aperture function into a finite sum of complex Gaussian functions. Numerical simulation results show the propagation properties and focusing properties.

2. Mathematical description of the beam generated by Gaussian mirror resonator

A Gaussian mirror is used as the output coupler in a laser resonator, which has a circularly symmetric reflectance profile. The field distribution over the output plane of the Gaussian mirror resonator can be expressed as [7]

$$U(\rho)=A_0\exp(-\rho^2/w_0{}^2-i\Theta)[1-K\exp(-2\beta\rho^2/w_0{}^2)]^{1/2}, \tag{1}$$

where A_0 represents the amplitude of a conventional Gaussian beam, ρ the radial coordinate, K the central reflectivity of the mirror, w_0 the beam waist, $\beta=(w_0/w_c)^2$, where w_c is defined as the mirror spot size at which the reflectivity is reduced to $\exp(-2)$ of its peak value, and $\Theta=\Theta(\rho)$ is the phase factor, which determines the wave-front curvature of the incident beam. In this paper, it is assumed as $\Theta=0$. Using the binomial expansion of $(1-x)^{1/2}(|x|\le 1)$, Eq. (1) can be re-expressed as

$$U(\rho)=\sum_{m=0}^{\infty}A_m\exp\left[-(2m\beta+1)\frac{\rho^2}{w_0^2}\right]=\sum_{m=0}^{\infty}A_m\exp\left[-\frac{\rho^2}{(w_0)_m^2}\right], \tag{2}$$

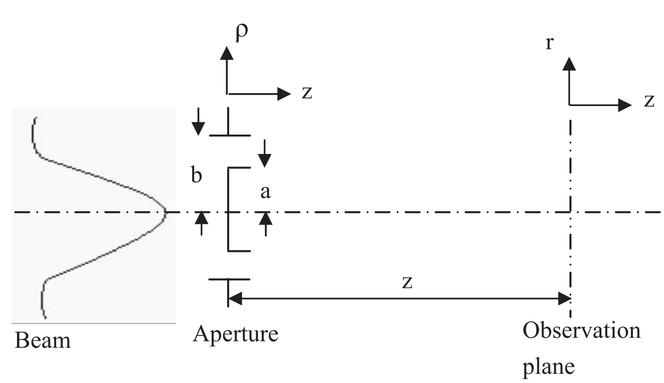

Fig. 1. Analysis model of the beam generated by a Gaussian mirror resonator passing through a paraxial annular aperture.

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where $A_m = \alpha_m A_0$, $\alpha_0 = 1$, $\alpha_1 = -(K/2)$,

$$\alpha_m = -\frac{(2m-3)(2m-5)\ldots(3)(1)}{m!}\left(\frac{K}{2}\right)^m, \quad (m \geq 2). \tag{3}$$

and

$$(w_0)_m = \frac{w_0}{(2m\beta+1)^{1/2}}, \quad (m = 0,1,2,\ldots). \tag{4}$$

Eq. (2) indicates that the beam generated by Gaussian mirror resonator is a function of the radial coordinate ρ, and that it can be decomposed into a linear combination of lowest-order Gaussian beams.

Table 1
The expansion coefficients A_h, A_g and the Gaussian coefficients B_h, B_g.

	A_h or A_g	B_h or B_g
1	11.428+0.95175i	4.0697+0.22726i
2	0.06002−0.08013i	1.1531−20.933i
3	−4.2743−8.5562i	4.4608+5.1268i
4	1.6576+2.7015i	4.3521+14.997i
5	−5.0418+3.2488i	4.5443+10.003i
6	1.1227−0.68854i	3.8478+20.078i
7	−1.0106−0.26955i	2.5280−10.310i
8	−2.5974+3.2202i	3.3197−4.8008i
9	−0.14840−0.31193i	1.9002−15.820i
10	−0.20850−0.23851i	2.6340+25.009i

3. Analysis model of the propagation process

Using the Collins formula [10], the propagation of a beam through a paraxial *ABCD* optical system with an annular aperture can be described as

$$U(r,\theta,z) = -\frac{i\exp(ikz)}{\lambda B}\int_0^{2\pi}\int_a^b U(\rho,\phi,0) \times\exp\left\{\frac{ik}{2B}\left[A\rho^2 - 2r\rho\cos(\theta-\phi) + Dr^2\right]\right\}\rho d\rho d\phi, \tag{5}$$

where r, θ and ρ, ϕ are the radial and azimuthal angle coordinates in the observation and the aperture planes, respectively (as shown in Fig. 1). $k = 2\pi/\lambda$ is the wave number, λ the wavelength A,B,C,D are the transfer matrix elements of the paraxial system, as well as a, b are respectively the inner radius and outer radius of the annular aperture.

Using the integral formula $\frac{1}{2\pi}\int_0^{2\pi} exp\left[\frac{ik}{B} r\rho\cos(\phi-\theta)\right]d\phi = J_0\left(\frac{k}{B}r\rho\right)$, where J_0 is the zeroth-order of the first kind Bessel function, Eq. (5) can be simplified as

$$U(r,\theta,z) = -\frac{2\pi i\exp(ikz)}{\lambda B}exp\left(\frac{ikDr^2}{2B}\right)\int_a^b U(\rho,\phi,0)\exp\left(\frac{ikA}{2B}\rho^2\right) \times J_0\left(\frac{kr}{B}\rho\right)\rho d\rho. \tag{6}$$

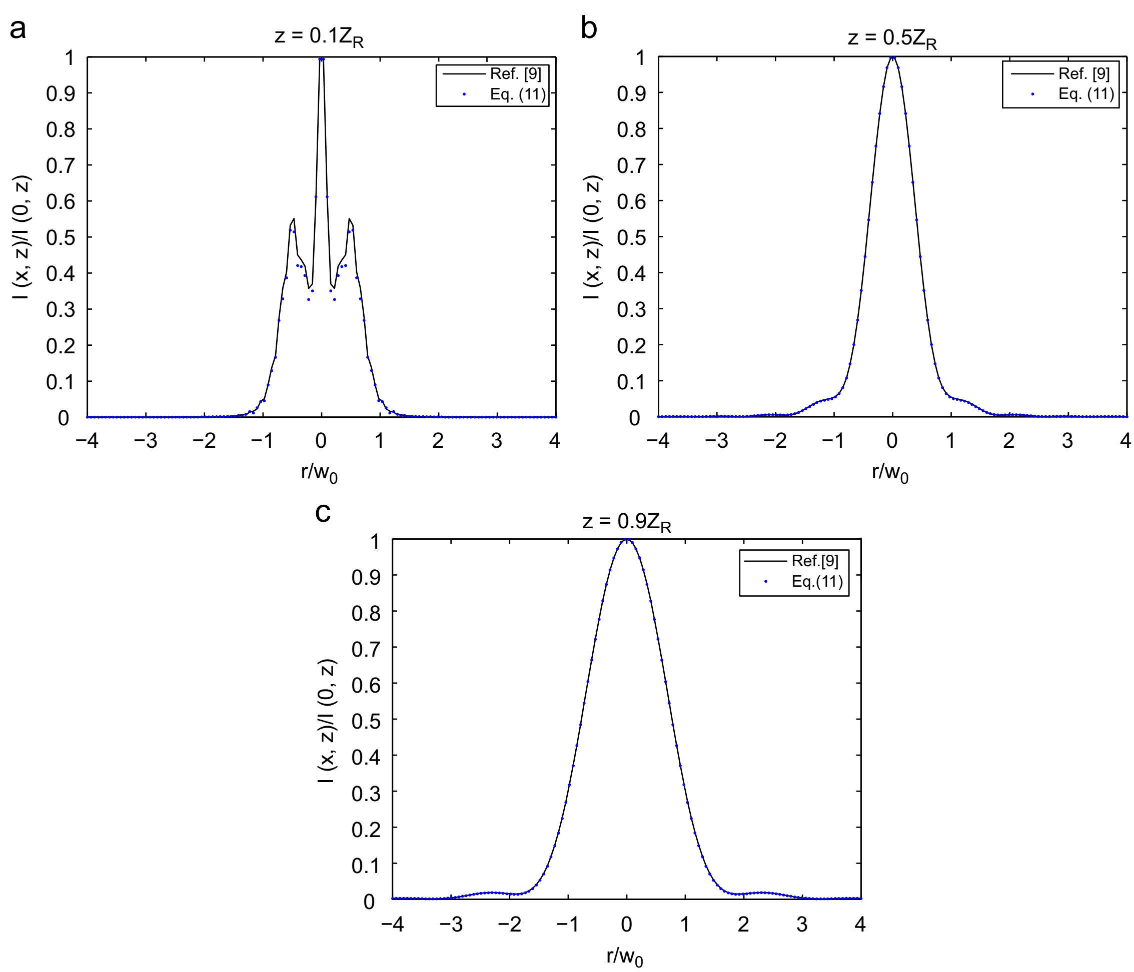

Fig. 2. Relative intensity distribution of the beam passing through a circular aperture in free space, where $Z_R = \pi w_0^2/\lambda$ and the parameters are a=0, b=2 × 790λ, K=0.7, β=1 and w_0=b.