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The M^2 factor of partially coherent ultrashort pulsed beams

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Abstract

Starting from the space-time Wigner distribution function and taking the Gaussian Schell-model pulsed beam as a typical example, the M^2 factor of partially coherent ultrashort pulsed beams is studied. It is shown that the M^2 factor increases with increasing bandwidth and decreases with increasing spatial correlation. Furthermore, for chirped pulse, the M^2 factor increases as the chirp parameter increases.

Keywords: M^2 factor; Space-time Wigner distribution function; Gaussian Schell-model pulsed beam

1. Introduction

In recent years ultrashort pulsed laser techniques have been developed rapidly. The space-time Wigner distribution function (STWDF) has been established and applied to the study of spatiotemporal characteristics of femtosecond pulses [1–3]. The second-order moments method has been extended to deal with pulsed light beams [4–7]. Using the spatiotemporal tensorial method in Ref. [4], the propagation law for the second-order moments of quasi-monochromatic pulses was derived, but for ultrashort pulses with broadband the quasi-monochromatic condition is not fulfilled [5]. Based on the Fourier transform method, the propagation invariant of pulsed beams was derived in Ref. [6]. It was revealed that the time-averaged spatial second-order moments follow the *ABCD* law. On the basis of the STWDF, the M^2 factor of partially coherent ultrashort pulsed beams was expressed in terms of the second-order moments matrix elements in Ref. [7]. The purpose of this paper is to study the M^2 factor of partially coherent ultrashort pulses and the dependence of the M^2 factor on the beam parameters. Starting from the STWDF and taking Gaussian Schell-model (GSM) pulsed beam as a typical example, an explicit expression for the M^2 factor of GSM pulsed beam is derived, which depends on the pulse bandwidth, spatial correlation degree and average wavelength. Furthermore, the M^2 factor of chirped GSM pulsed beams is additionally dependent on the chirped parameter. The analytical results are illustrated with numerical examples.

2. Basic theory

Let us consider a general light pulsed beam with electric field $E(\mathbf{r},z,t)$ in the space-time domain, where $\mathbf{r} = (x, y)$ is the spatial vector at the z plane, and t is the temporal variable. Based on the Fourier transform method, in the frequency domain we have

$$\hat{E}(\mathbf{r}, z, \omega) = \int E(r, z, t)\exp(-\mathrm{i}\omega t)\mathrm{d}t. \tag{1}$$

The integration extends from $-\infty$ to $+\infty$ (unless otherwise stated).

For polychromatic pulses the STWDF is written as [1]

$$W^{ST}(\mathbf{r}, \mathbf{u}; t, \omega) = \int \frac{1}{\lambda^2}\left\langle \hat{E}\left(\mathbf{r} + \mathbf{r}'/2, \omega + \omega'/2\right) \times \hat{E}^*\left(\mathbf{r} - \mathbf{r}'/2, \omega - \omega'/2\right)\right\rangle \times \exp\left[-\mathrm{i}\left(\frac{2\pi\mathbf{u}^{\mathrm{T}}\mathbf{r}'}{\lambda} - \omega' t\right)\right]\mathrm{d}\mathbf{r}'\mathrm{d}\omega', \tag{2}$$

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where $\mathbf{u}=(u_x, u_y)$ is the propagation angle vector, $<>$ and $\mathbf{T}$ denote the ensemble average and the operation of transposition, respectively.

If the time-averaged process is considered, i.e.

$$W^S(\mathbf{r},\mathbf{u};\omega)=\int W^{ST}(\mathbf{r},\mathbf{u};t,\omega)\mathrm{d}t, \tag{3}$$

thus the integral of the STWDF with t leads to

$$W^S(\mathbf{r},\mathbf{u};\omega)=\int \frac{1}{\lambda^2}\left\langle \hat{E}(\mathbf{r}+\mathbf{r}'/2,\omega)\hat{E}^*(\mathbf{r}-\mathbf{r}'/2,\omega)\right\rangle \times \exp\left(-\mathrm{i}\frac{2\pi}{\lambda}\mathbf{u}^{\mathrm{T}}\mathbf{r}'\right)\mathrm{d}\mathbf{r}', \tag{4}$$

where the relation

$$(2\pi)^{-1}\int \exp[-\mathrm{i}\omega' t]\mathrm{d}t=\delta(\omega') \tag{5}$$

has been used, and $\delta(\cdot)$ denotes the Dirac δ function, * is the complex conjugate.

A paraxial non-dispersive optic system can be characterized by an $\boldsymbol{ABCD}$ matrix, namely,

$$\begin{bmatrix}\mathbf{r}_o\\ \mathbf{u}_o\end{bmatrix}=J\begin{bmatrix}\mathbf{r}_i\\ \mathbf{u}_i\end{bmatrix},\quad J=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{C} & \boldsymbol{D}\end{bmatrix}, \tag{6}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are all 2×2 matrices that are not related to wavelength λ. J is a symplectic matrix and has a unit determinant. W_i^S at the input plane and W_o^S at the output plane are related by [7]

$$W_i^S(\mathbf{r},\mathbf{u};\omega)=W_o^S(\boldsymbol{A}\mathbf{r}+\boldsymbol{B}\mathbf{u},\boldsymbol{C}\mathbf{r}+\boldsymbol{D}\mathbf{u};\omega). \tag{7}$$

By employing Eq. (4) one can define the second-order moments matrix m of partially coherent ultrashort pulsed beams as

$$\boldsymbol{m}=\begin{bmatrix}\boldsymbol{m}_{rr} & \boldsymbol{m}_{ru}\\ \boldsymbol{m}_{ur} & \boldsymbol{m}_{uu}\end{bmatrix}=\int\begin{bmatrix}\mathbf{r}\mathbf{r}^{\mathrm{T}} & \mathbf{r}\mathbf{u}^{\mathrm{T}}\\ \mathbf{u}\mathbf{r}^{\mathrm{T}} & \mathbf{u}\mathbf{u}^{\mathrm{T}}\end{bmatrix}W^S(\mathbf{r},\mathbf{u};\omega)\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{u}\,\mathrm{d}\omega, \tag{8}$$

and assume that

$$\int W^S(\mathbf{r},\mathbf{u};\omega)\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{u}\,\mathrm{d}\omega=1. \tag{9}$$

m is a real positive definite and symmetric 4×4 matrix [7].

The expression of the M^2 factor was given by Eq. (23) in Ref. [7], which reads as

$$M^4=(4\pi/\bar{\lambda})^2\mathrm{tr}\left(m_{\mathrm{rr}}m_{\mathrm{uu}}-m_{\mathrm{ru}}^2\right), \tag{10}$$

where tr denotes the trace of the matrix, and

$$\bar{\lambda}=\int \lambda W^S(\mathbf{r},\mathbf{u};\omega)\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{u}\,\mathrm{d}\omega \tag{11}$$

is the average wavelength. The above results are valid for both spatially fully and partially coherent beams in a time-averaged process.

3. M^2 factor of Gaussian Schell-model and chirped Gaussian Schell-model pulsed beams

Consider the propagation of a rotationally symmetric GSM pulsed beams through a paraxial non-dispersive optic system, whose cross-spectral density function at the input plane $z=0$ reads as

$$\left\langle \hat{E}(\mathbf{r}_{i1},0;\omega)\hat{E}^*(\mathbf{r}_{i2},0;\omega)\right\rangle = S^{(0)}(\omega)\exp\left(-\frac{|\mathbf{r}_{i1}-\mathbf{r}_{i2}|^2}{2\sigma_0^2}\right)\times\exp\left(-\frac{\mathbf{r}_{i1}^{\mathrm{T}}\mathbf{r}_{i1}+\mathbf{r}_{i2}^{\mathrm{T}}\mathbf{r}_{i2}}{w_0^2}\right), \tag{12}$$

where σ_0 and w_0 are the correlation length and waist width, respectively, and $S^{(0)}(\omega)$ stands for the power spectrum (spectral intensity) at the input plane $z=0$.

The Fourier spectrum of ultrashort pulsed beams with an arbitrary temporal profile $A_0(t)$ at the input plane $z=0$ reads as

$$f(\omega)=(2\pi)^{-1/2}\int A_0(t)\exp(-\mathrm{i}\omega t)\mathrm{d}t, \tag{13}$$

the initial power spectrum is expressed as

$$S^{(0)}(\omega)=|f(\omega)|^2. \tag{14}$$

The substitution from Eq. (12) into (4) yields

$$W_i^S(\mathbf{r},\mathbf{u};\omega)=S^{(0)}(\omega)\frac{4}{\lambda^2}\left(\frac{\sigma_0^2}{w_0^2+\sigma_0^2}\right)\times\exp\left[-\frac{2\pi^2\sigma_0^2w_0^2}{\lambda^2(\sigma_0^2+w_0^2)}\mathbf{u}^{\mathrm{T}}\mathbf{u}\right]\times\exp\left(-\frac{2}{w_0^2}\mathbf{r}^{\mathrm{T}}\mathbf{r}\right), \tag{15}$$

where w_0 and σ_0 are assumed to be independent of the frequency ω, and

$$\int S^{(0)}(\omega)\mathrm{d}\omega=1. \tag{16}$$

The substitution from Eqs. (15) and (8) into Eq. (10) yields

$$M^4=(1+\alpha^{-2})\left[1+\frac{(\Delta\lambda)^2}{\bar{\lambda}^2}\right], \tag{17}$$

where

$$(\Delta\lambda)^2=\overline{\lambda^2}-\bar{\lambda}^2, \tag{18}$$

$$\overline{\lambda^2}=\int \lambda^2 W^S(\mathbf{r},\mathbf{u};\omega)\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{u}\,\mathrm{d}\omega, \tag{19}$$

and $\alpha=\sigma_0/w_0$ denotes the spatial correlation degree.

Two special cases of Eq. (17) are of interest. (1) If the spatially fully coherent Gaussian pulsed beams are considered, i.e. the spatial correlation degree $\alpha\to\infty$, we obtain $M^2=\left[1+(\Delta\lambda/\bar{\lambda})^2\right]^{1/2}$ [6]; (2) For quasi-monochromatic GSM pulsed beams $(\Delta\lambda)^2/\bar{\lambda}^2\to 0$, we have $M^2=(1+\alpha^{-2})^{1/2}$ [8].