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[INVITED] Soliton propagation through nanoscale waveguides in optical metamaterials ☆

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ABSTRACT

This paper studies the dynamics of soliton propagation through optical metamaterials. The proposed model will be studied with five forms of nonlinearity. They are Kerr law, power law, parabolic law, dual-power law and log-law. The integration scheme that will be adopted is the method of undetermined coefficients. Bright, dark and singular soliton solutions will be obtained. The essential conditions for the existence of these solitons will naturally emerge.

1. Introduction

The theory of solitons in optical fibers and optical metamaterials is a very fascinating area of research in nonlinear optics [1–25]. Optical metamaterials possess both negative permittivity and negative permeability that cannot be found in nature; but can be engineered by using advanced processing technology [17]. This material has been fabricated using nano-fabrication technology by several research groups [11,17]. They manipulated the periodic structure of photonic crystal as well as resonant ring for negative permeability [11,17]. Recently, by using metamaterials, Shalaev and others demonstrated optical waveguides in visible and infrared regions [17]. One inherent property of optical metamaterials in optical frequency is its loss. Different waveguide structures were proposed using optical metamaterials [17]. As long as optical wave is guided, soliton pulses can evolve owing to delicate balance between dispersion and nonlinearity. However it is always a challenge to compensate for the loss when engineering these types of waveguides using metamaterials. The theoretical results showed that metamaterials enhance nonlinearity by confining electrical field in a small region that allows more light–matter interaction [11,17,19,20]. In metamaterials, linear and nonlinear coefficients of the propagation equation can be tuned to achieve any combination of signs that is not possible in regular materials. These properties of metamaterials lead to improved propagation of a wider variety of solitary waves, efficient phase-matching and modulational instability [12,19,20]. Numerical as well as analytical results of soliton propagation in several nanoscale optical waveguides were reported by several authors [12,19,20]. Earlier results reveal that similar regular (positive indexed) dielectric material dispersion plays a pivotal role in supporting short duration soliton pulses. Optical waveguides with selected wavelengths can be implemented in photonic crystal partially filled with gold and nanoparticles. Recently, theoretical results are reportedfor *Y*-splitter and bend waveguide structures [11].

The dynamics of soliton propagation through these optical metamaterials is governed by the nonlinear Schrödinger's equation (NLSE) with a few perturbation terms. This model was first reported during 2011 [21]. With the advent of such a model, a plethora of results have been reported. The integrability aspect of this model was studied with various forms of nonlinearity. The integration tools that were applied are simplest equation approach, functional variable method, first integral scheme, Kudryashov's method, trial solutions approach, *F*-expansion scheme and others [5–9,13]. These algorithms yielded solitons, shock waves and other solutions to the model that appeared with several integrability conditions. In addition to these exact soliton solutions, very recently semi-inverse variational principle was applied to extract bright and exotic soliton solutions to the model [23].

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These are analytical soliton solutions although they are not exact. This paper will apply the method of undetermined coefficients that is otherwise conveniently known as ansatz scheme, to retrieve exact soliton solutions. Bright, dark and singular soliton solutions will be recovered that will appear with essential integrability conditions which stems out from the solution structure of the model.

2. Governing equation and mathematical analysis

The dynamics of solitons in optical metamaterials is governed by the nonlinear Schrödinger's equation (NLSE) which in the dimensionless form is given by [20]

$$iq_t + aq_{xx} + F\left(|q|^2\right)q = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2 q\right)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \tag{1}$$

Eq. (1) is the NLSE that is studied in the context of metamaterials. Here in (1), a and b are the group velocity dispersion and the self-phase modulation terms respectively. This pair produces the delicate balance between dispersion and nonlinearity that accounts for the formation of the stable solitons. On the right-hand side λ represents the self-steepening term in order to avoid the formation of shocks and ν is the nonlinear dispersion, while α represents the inter-modal dispersion. This arises from the fact that group velocity of light in multi-mode fibers depends on chromatic dispersion as well as the propagation mode involved. Next, θ_j for $j = 1, 2, 3$ are the perturbation terms that appear in the context of metamaterials [1,5–9,13]. Finally, the independent variables are x and t that represent spatial and temporal variables respectively with the dependent variable $q(x, t)$ being the complex-valued wave profile.

The real-valued algebraic functional F must possess smoothness of the complex-valued function $F(|q|^2)q: C \mapsto C$. Treating the complex plane C as two-dimensional linear space R^2, the function $F(|q|^2)q$ is k times continuously differentiable provided

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n, n) \times (-m, m); R^2\right). \tag{2}$$

In order to start with the analysis of (1), the starting hypothesis is

$$q(x, t) = P(x, t)e^{i\phi}, \tag{3}$$

In (2), $P(x, t)$ represents amplitude portion of the wave while $\phi(x, t)$ is the phase component that is given by

$$\phi = -\kappa x + \omega t + \theta. \tag{4}$$

where κ gives the soliton frequency and ω being the soliton wave number while θ represent the phase constant. After substituting (3) into (1) and decomposing into real and imaginary parts lead to

$$\left(\omega + \alpha\kappa + a\kappa^2\right)P - a\frac{\partial^2 P}{\partial x^2} + 6\theta_1 P\left(\frac{\partial P}{\partial x}\right)^2 - P^3\left\{b - \lambda\kappa + \kappa^2\left(\theta_1 + \theta_2 + \theta_3\right)\right\} + P^2\frac{\partial^2 P}{\partial x^2}\left(3\theta_1 + \theta_2 + \theta_3\right) = 0 \tag{5}$$

and

$$\frac{\partial P}{\partial t} - (\alpha + 2a\kappa)\frac{\partial P}{\partial x} = \left(3\lambda + 2\nu - 6\theta_1\kappa - 2\theta_2\kappa + 2\theta_3\kappa\right)P^2\frac{\partial P}{\partial x} \tag{6}$$

respectively. The imaginary part equation (6) implies the relations

$$v = -\alpha - 2a\kappa \tag{7}$$

and

$$3\lambda + 2\nu = 2\kappa\left(3\theta_1 + \theta_2 - \theta_3\right). \tag{8}$$

This follows from the fact that the amplitude portion $P(x, t)$ can be written in terms of the wave variable $g(x - vt)$ with ν being the speed of the wave. The two relations (7) and (8) are obtained by setting the coefficients of linearly independent functions from (6) to zero. These two expressions serve as the existence condition for the solitons that is commonly referred to as constraint relation.

The speed of the soliton stays the same for all laws of nonlinearity, namely for all forms of the functional F introduced in (1) and for all kinds of solitons. The constraint relation (8) however modifies with power and dual-power laws. It is the real part equation that will be further analyzed in detail for various nonlinear forms of F in the following sections.

3. Kerr law

This law is also known as the cubic nonlinearity and is considered to be the simplest known form of nonlinearity. Most optical fibers that are commercially available nowadays obey this Kerr law of nonlinearity. Therefore, in this first section the attention will be on optical metamaterials with cubic nonlinearity. In this case $F(u) = bu$ for some non-zero constant b [4]. Therefore, the governing equation given by (1) with Kerr law nonlinearity reduces to [5]

$$iq_t + aq_{xx} + b|q|^2 q = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2 q\right)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \tag{9}$$

For Kerr law nonlinearity the results of bright, dark and singular soliton have been already reported in the past [5,6]. Therefore, this section will just list the results from these earlier published results [5,6]. It is only the singular solitons of second type that will be derived in detail.

3.1. Bright solitons

For Kerr law nonlinear medium, bright 1-soliton solution in optical metamaterials is given by [5]

$$q(x, t) = A\,\mathrm{sech}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{10}$$

where A is the amplitude and B is the inverse width of the soliton. The relation between amplitude and width is given by

$$\left(b - \lambda\kappa - 5\theta_1\kappa^2\right)A^2 - 3\theta_1 A^2 B^2 - 2aB^2 = 0. \tag{11}$$

The wave number is

$$\omega = aB^2 - a\kappa^2 - \alpha\kappa \tag{12}$$

and the additional constraint condition is

$$6\theta_1 + \theta_2 + \theta_3 = 0. \tag{13}$$

3.2. Dark solitons

For Kerr law, dark soliton solution is given by [5]

$$q(x, t) = A\tanh[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)}. \tag{14}$$

In this case, the parameters A and B are referred to as free parameters and these are connected as

$$A^2\left(b - \lambda\kappa - 5\theta_1\kappa^2\right) + 6\theta_1 A^2 B^2 + 2aB^2 = 0 \tag{15}$$

and the wave number is