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Dependence of the beam wander of an airy beam on its kurtosis parameter in a turbulent atmosphere

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ABSTRACT

Beam wander, which is affected by many factors, is an important characteristic of laser beams in a turbulent atmosphere. In the present letter the influence of an Airy beam's kurtosis parameter on its beam wander has been studied. The interesting result is that the beam wander of an Airy beam has a simple and fixed rule versus its kurtosis parameter, regardless of the propagation distance, characteristic scales of the beam, inner scales of turbulence, wavelength and the initial coherence length. It can be used in the control of beam wander for Airy beam in practice, such as in the free-space-optical communication and laser defense.

1. Introduction

Non-diffracting beams are characterized by the fact that the field factors in transversely and longitudinally dependent parts behave in such a way that the intensity of the longitudinal part is independent of the propagation distance. Perhaps the best known example of such nondiffracting beam is the so-called Bessel beam as first suggested and observed by Durnin et al. in 1987 [1]. In the last few decades, nondiffracting beams have attracted considerable attention because of their intriguing properties and their potential applications. More recently, a specific type of non-diffracting beam, namely, the self-accelerating Airy beam was introduced [2] and was experimentally demonstrated in optics by Siviloglou and Christodoulies [3]. In the past years, Airy beams have attracted a great deal of interest due to their unique properties such as propagation along parabolic trajectories (self accelerating) [3], nondiffracting and self-healing [4,5].

It is well known that a laser beam will experience random perturbations in the refractive index when it propagates through a turbulent atmosphere. As a result, the beam profile at the receiver randomly moves off the boresight over short time periods and the instantaneous center of the beam is therefore randomly displaced in the receiver plane, producing what is called beam wander [6]. Beam wander is an important characteristic of a laser beam, which determines its utility for practical applications such as uninterrupted laser tracking and free-space-optical communication [7]. In the past five decades, beam wander for a variety of beams with different shapes has been studied [8–17]. These works show that the beam wander can be influenced by many factors, including the initial spatial coherence length [8–11], amplitude factor [12], source size [13,14], degree of polarization [15], topological charge [16], divergence and convergence [17], and the turbulence [8–17]. Moreover, the rules relating the beam wander to these factors are complex and diverse. In these investigations one can see that there are many terms to consider if people want to control the effect of the beam wander. Obviously, it is very unfavorable to consider so many factors to manipulate this effect in practical applications. In this letter we investigate the influence of an Airy beam's kurtosis parameter on its beam wander and find that there is a simple and fixed relationship between the beam wander of an Airy beam and its kurtosis parameter regardless of the variation of propagation distance, characteristic scales of the Airy beam, inner scales of turbulence, the initial spatially coherent length and wave length. This study may provide a new theoretical basis to effectively control the beam wander of an Airy beam.

From the existing results we can infer that the beam shape can influence the beam wander of a laser beam. The kurtosis parameter K, which relates to the second-order and fourth-order intensity moments, is a parameter often used to describe the flatness of the beam's intensity distribution [18,19–23]. As yet the kurtosis parameters of various laser beams such as cosh-Gauss beam, Hermite–Gaussian beam, flattened Gaussian beam, super-Gaussian beam and

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fundamental Gaussian beam have been derived [20–23]. The kurtosis parameters are $1<K<3$[20], $1.5<K<3$ [21], $1.85<K<3$ [22], $2<K<3$ [23] and 3 for cosh-Gauss beam, Hermite–Gaussian beam, flattened Gaussian beam, super-Gaussian beam, and fundamental Gaussian beam respectively in the initial plane under a Cartesian coordinate system. More recently, it has been shown that the kurtosis parameter is $3<K<15$ for an Airy beam [18]. It can be seen that the diversification in beam's shape of an Airy beam is more significant compared with those beams mentioned above. In this research, we will report the relation between the kurtosis parameter and the beam wander of an Airy beam in a turbulent atmosphere.

2. Formulation

In this section, an analytic formulation for the beam wander of an Airy beam passing through a turbulent atmosphere is presented. An Airy beam in Cartesian coordinates at initial plane is given as [2]

$$u_0(x_0,y_0,0)=Ai\left(\frac{x_0}{w_0}\right)Ai\left(\frac{y_0}{w_0}\right)\exp\left[\frac{a(x_0+y_0)}{w_0}\right], \tag{1}$$

Here $Ai(\ldots)$ is the Airy function, (x_0,y_0) are the transverse coordinates at the initial plane, w_0 and a are characteristic scales and exponential truncation factor, respectively. Based on the methods of statistical optics, we have investigated arbitrary moments of an Airy beam in a turbulent atmosphere. Moreover, we have studied the evolution of the kurtosis parameter, K, of an Airy beam in a turbulent atmosphere exhaustively. Previously we have obtained the kurtosis parameter of an Airy beam in the initial plane as [18]

$$K=\frac{3(5+16a^3+64a^6)}{(1+8a^3)^2}. \tag{2}$$

Generally, beam wander is characterized by the random displacement r_c of the instantaneous center of the beam as it propagates through a turbulent atmosphere. A model of beam wander that is valid under all turbulence conditions is given by Andrews and Phillips as [6]

$$\langle r_c^2\rangle=4\pi k^2W_{FS}^2\int_0^L\int_0^\infty \kappa\Phi_n(\kappa)\exp\left(-\kappa^2W_{LT}^2\right)\left\{1-\exp\left[-\frac{2L^2\kappa^2(1-z/L)^2}{k^2W_{FS}^2}\right]\right\}d\kappa dz, \tag{3}$$

where κ is the spatial frequency, $k=2\pi/\lambda$ is the wavenumber, with λ being the wavelength, L is the total propagation path length, and z is the distance of an intercept point from the input plane at $z=0$. $\Phi_n(\kappa)$ is the atmospheric spectrum and it is assumed to be the Tatarskii spectrum in this study, i.e.

$$\Phi_n(k)=0.033C_n^2\kappa^{-11/3}\exp\left(-\frac{\kappa^2}{\kappa_m^2}\right), \tag{4}$$

where $\kappa_m=5.92/l_0$ (l_0 being the inner scale of turbulence) and C_n^2 is the structure constant. W_{LT} is the long-term width which can be defined by [15]

$$W_{LT}=\sqrt{\frac{2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[(x-x_c)^2+(y-y_c)^2\right]\langle I(x,y,z)\rangle dx\,dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle I(x,y,z)\rangle\,dx\,dy}}, \tag{5}$$

where $\langle I(x,y,z)\rangle$ is the average intensity of the Airy beam and $x_c=y_c=(4a^3-1)w_0/4a$ is the centroid of the Airy beam [18,24]. From Eq. (5) the long-term width of the Airy beam can be expressed as

$$W_{LT}=2w_0\sqrt{\frac{8a^3+1}{8a^2}+\frac{z^2}{4ak^2w_0^4}+\frac{2z^2T}{k^2w_0^2}}, \tag{6}$$

where

$$T=0.033\Gamma(7/6)\pi^2k^2C_n^2\kappa_m^{1/3}z. \tag{7}$$

From Eq. (6) one can readily obtain the beam width W_{FS} by setting $C_n^2=0$ and $z=L$. Substituting (7), (6) and (4) into Eq. (3) gives

$$\langle r_c^2\rangle=0.066\pi k^2W_{FS}^2C_n^2\Gamma(-5/6)\int_0^L\left\{\left(\frac{1}{\kappa_m^2}+W_{LT}^2\right)^{5/6}-\left[\frac{2L^2(1-z/L)^2}{k^2W_{FS}^2}+\frac{1}{\kappa_m^2}+W_{LT}^2\right]^{5/6}\right\}dz. \tag{8}$$

Eq. (8) is the main analyzed result of this letter. It shows that the beam wander of an Airy beam varies with the changes in the propagation distance, wavenumber, refractive index structure constant, inner scale of turbulence, long-term beam width and the beam width without turbulence at the receiver plane.

3. Numerical results and analysis

It can be seen from Eq. (2) that the kurtosis parameter is 15 when $a=0$ and it converges to 3 if a is large enough. In practice, Airy beams have multiple maxima and minima and are not a single peak when the kurtosis parameter is large. However, the shape of an Airy beam resembles a Gaussian distribution when $K\to3$. Because the Airy beam is symmetric, only a one-dimensional kurtosis parameter is discussed here. The beam's flatness of an Airy beam for different kurtosis parameters is illustrated in Fig. 1.

From Fig. 1 one can seen that the beam shape is more diverse with an increase of the kurtosis parameter. Fig. 1(a) demonstrates that there is only one sidelobe behind the main spot when $K=4.3$. As the number of lateral petals increases, the kurtosis parameter also increases.

The dimensionless quantity $B_W=\langle r_c^2\rangle/W_{LT}^2$ is more informative than $\langle r_c^2\rangle$ about the practical significance of the beam wander effect. We use B_W to investigate the beam wander of an Airy beam by numerical calculation. To see the relation between the beam's wander and its kurtosis parameter, the evolution of the wander with its kurtosis parameter under different turbulence conditions is shown in Figs. 2–5.

Fig. 2 displays the beam wander of an Airy beam as a function of the kurtosis for different propagation distances and the structure constant of the turbulence. It shows that the beam wander is

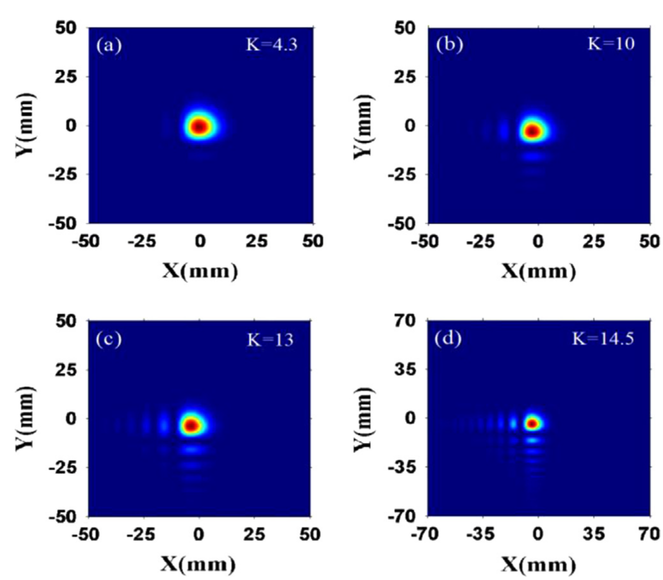

Fig. 1. Beam flatness of Airy beam for four values of the kurtosis parameter K.