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Time-dependent scintillations of pulsed Gaussian-beam waves propagating in generalized atmospheric turbulence

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ABSTRACT

Expressions for both the temporal first- and second-order intensity moments of pulsed Gaussian-beam waves passing through generalized atmospheric turbulence are derived under the near- and far-field approximations, respectively. With the help of these expressions, the time-dependent scintillation behavior of optical pulses during propagation in generalized atmospheric turbulence is examined by numerical calculations. The effects that the spectral index of the spatial power spectrum of refractive-index fluctuations has on the temporal dependence of pulse scintillations are analyzed under the condition that the generalized plane-wave Rytov variance is specified as a constant for various spectral indices. It is shown that both the near- and far-field scintillations of an optical pulse are less dependent on time as the spectral index becomes smaller, indicating that there does exist a significant difference between the time-dependent scintillation behavior of an optical pulse in non-Kolmogorov turbulence and that in the Kolmogorov one. The obtained results are helpful for understanding the time-dependent scintillations of optical pulses propagating in generalized atmospheric turbulence and hence are useful for practical applications.

1. Introduction

Scintillations of optical waves in atmospheric turbulence have attracted much attention over many years due to the reason that many practical applications, e.g., free-space optical communications, remote sensing and laser radar, involve beam propagation in the atmosphere. Up to now, most of the studies concerning turbulence-induced optical scintillations have considered the beams as a continuous optical wave [1–3], which can be viewed as a stationary optical field [4], at least in a wide sense. For a stationary optical field, both the second- and fourth-order statistics are time-independent. However, pulsed beam waves, which essentially belong to non-stationary optical fields [5], have also been widely employed in optical engineering systems. Theoretically, both the second- and fourth-order statistics of non-stationary optical fields in atmospheric turbulence should be time-dependent. As a result, Kelly and Andrews [6] defined the temporal scintillation index to characterize the turbulence-induced time-dependent intensity fluctuations of pulsed beam waves, and examined the time-dependent scintillation behavior of optical pulses passing through atmospheric turbulence obeying the von Kármán spectrum.

Although the Kolmogorov theory for atmospheric turbulence, which assumes that the structure function obeys a 2/3 power law within the inertial sub-range [7], has been widely considered to be in good agreement with the experimental data in the past, recently it has been shown that this theory cannot accurately describe the turbulence statistics in certain portions of the atmosphere, e.g., the upper troposphere and stratosphere [8]. Hence, the so-called non-Kolmogorov theory [2,8] has been developed to model the spatial power spectrum of refractive-index fluctuations incompatible with the Kolmogorov theory. It is noted that the aforementioned von Kármán spectrum, including both the inner- and outer-scale parameters of turbulence, is actually derived based on the Kolmogorov theory. Recently, generalized spatial power spectra with an arbitrary spectral index have been developed to describe the refractive-index fluctuations of non-Kolmogorov turbulence [2,8–10]. In fact, the non-Kolmogorov theory can be viewed as a generalization of the Kolmogorov one. Therefore, non-Kolmogorov turbulence is also referred to as generalized atmospheric turbulence [9].

The spectral index of the spatial power spectrum of refractive-index fluctuations has an important impact on the scintillations of continuous Gaussian-beam waves and on the temporal broadening of pulsed Gaussian-beam waves in generalized atmospheric turbulence

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[2,11]. Consequently, we conjecture that the spectral index also significantly affects the time-dependent scintillation behavior of pulsed Gaussian-beam waves traveling in generalized atmospheric turbulence. However, to the best of our knowledge, there have been no reports on this issue. To understand the dependence of pulse scintillations on the spectral index of the spatial power spectrum of refractive-index fluctuations, in this paper, we first develop mathematical models for the time-dependent scintillations of an optical pulse propagating in generalized atmospheric turbulence and then examine the effects of the spectral index on the time-dependent scintillation behavior of the pulse by numerical calculations.

2. Theoretical formulations

We consider that a pulsed Gaussian-beam wave propagates along the positive z-axis in weak generalized atmospheric turbulence from the source plane $z=0$ to an observation plane $z=L$, and the pulse at the source plane is denoted by $P_i(t)=v_i(t)\exp(-i\omega_0 t)$ where $\omega_0=2\pi c/\lambda_0$ represents the carrier angular frequency with c being the speed of light and λ_0 the carrier wavelength; $v_i(t)=a_0\exp(-t^2/T_0^2)$ denotes the temporal pulse shape with a_0 being the peak amplitude and T_0 the temporal half-width. Following Kelly and Andrews [6], the scintillation index of a pulsed Gaussian-beam wave in atmospheric turbulence at time t, i.e., the so-called temporal scintillation index of optical pulses, is defined by

$$\sigma_I^2(\mathbf{r},L;t)=\frac{\langle I^2(\mathbf{r},L;t)\rangle}{\langle I(\mathbf{r},L;t)\rangle^2}-1, \tag{1}$$

where $\mathbf{r}$ is a position vector in the observation plane; L represents the propagation distance; $\langle I(\mathbf{r},L;t)\rangle$ and $\langle I^2(\mathbf{r},L;t)\rangle$ are the temporal first- and second-order intensity moments of the pulsed Gaussian-beam wave, respectively; the angle brackets denote an ensemble average. In what follows, we first develop the expressions for the temporal first-order intensity moment, and then formulate the temporal second-order one.

2.1. Temporal first-order intensity moment

Following an approach analogous to that of Kelly and Andrews [6], the first-order intensity moment of a pulsed Gaussian-beam wave in atmospheric turbulence at time t can be written as

$$\langle I(\mathbf{r},L;t)\rangle=\frac{a_0^2T_0^2}{4\pi}\int\int_{-\infty}^{\infty}\exp\left[-\frac{1}{4}(\omega_1^2+\omega_2^2)T_0^2\right]\exp[-i(\omega_1-\omega_2)t]\times\Gamma_2(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)\mathrm{d}\omega_1\,\mathrm{d}\omega_2 \tag{2}$$

with $\Gamma_2(\cdot)$ being the single-point, two-frequency mutual coherence function (MCF) given by

$$\begin{aligned}\Gamma_2(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)&=\langle u(\mathbf{r},L;\omega_0+\omega_1)u^*(\mathbf{r},L;\omega_0+\omega_2)\rangle\\&=\Gamma_2^{(0)}(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)\\&\quad\times M_2(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2),\end{aligned} \tag{3}$$

where $u(\mathbf{r},L;\omega_0+\omega_m)$ is the spectral component of frequency $\omega_0+\omega_m$ of the pulsed optical field in the presence of atmospheric turbulence ($m=1$, 2); the asterisk denotes the complex conjugate; $\Gamma_2^{(0)}(\cdot)$ represents the free-space single-point, two frequency MCF; $M_2(\cdot)$ stands for the factor due to the contribution from atmospheric turbulence. Note that $\Gamma_2^{(0)}(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)=<u_0(\mathbf{r},L;\omega_0+\omega_1)u_0^*(\mathbf{r},L;\omega_0+\omega_2)>$, where $u_0(\mathbf{r},L;\omega_0+\omega_m)$ is the spectral component of frequency $\omega_0+\omega_m$ of the pulsed optical field in free space ($m=1$, 2).

For a collimated pulsed Gaussian beam, under the near-field approximation, $\Gamma_2^{(0)}(\cdot)$ can be expressed in the form [7]

$$\Gamma_2^{(0)}(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)\cong\exp\left[i\tilde{\omega}_{12}\frac{L}{c}-\frac{2r^2}{W_0^2}\right], \tag{4}$$

where W_0 is the initial beam radius, $r=|\mathbf{r}|$, and $\tilde{\omega}_{12}=\omega_1-\omega_2$. On the other hand, under the far-field approximation, $\Gamma_2^{(0)}(\cdot)$ for a collimated pulsed Gaussian beam can be written as [7]

$$\begin{aligned}\Gamma_2^{(0)}(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)&\cong(\omega_0+\omega_1)(\omega_0+\omega_2)\left(\frac{W_0^2}{2Lc}\right)^2\exp\left[i\left(\frac{L}{c}+\frac{r^2}{2Lc}\right)\tilde{\omega}_{12}\right]\\&\times\exp\left[-\frac{1}{8}\left(\frac{W_0r}{Lc}\right)^2\tilde{\omega}_{12}^2-\frac{1}{2}\left(\frac{W_0r}{Lc}\right)^2(\omega_0+\omega_{12})^2\right],\end{aligned} \tag{5}$$

where $\omega_{12}=(\omega_1+\omega_2)/2$.

If the narrowband approximation is further assumed, under both the near- and far-field conditions, employing the Rytov perturbation method, it readily follows from Kelly and Andrews [6] and Young et al. [12] that

$$M_2(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)\cong\exp\left[-2\pi^2c^{-2}\tilde{\omega}_{12}^2L\int_0^{\infty}\mathrm{d}\kappa\,\kappa\Phi_n(\kappa)\right], \tag{6}$$

where $\Phi_n(\cdot)$ denotes the spatial power spectrum of refractive-index fluctuations. For generalized atmospheric turbulence, it has been shown that [10,11,13]

$$\Phi_n(\kappa)=A(\alpha)\tilde{C}_n^2\exp(-\kappa^2/\kappa_m^2)(\kappa^2+\kappa_0^2)^{-\alpha/2},\quad 3<\alpha<4, \tag{7}$$

where $\tilde{C}_n^2$ is a generalized refractive-index structure constant in units of $\mathrm{m}^{3-\alpha}$, $A(\alpha)=\Gamma(\alpha-1)\cos(\alpha\pi/2)/(4\pi^2)$, with $\Gamma(\bullet)$ being the gamma function $\kappa_0=2\pi/L_0$ with L_0 being the outer scale of turbulence, $\kappa_m=\beta/l_0$ with l_0 denoting the inner scale of turbulence and $\beta=[2\pi\Gamma(5-\alpha/2)A(\alpha)/3]^{1/(\alpha-5)}$.

Introducing Eq. (7) into Eq. (6) and employing the method for evaluating integrals based on the Mellin convolution theorem [14–17], one finds

$$M_2(\mathbf{r},L;\omega_0+\omega_1,\omega_0+\omega_2)=\exp(-Q_1\tilde{\omega}_{12}^2), \tag{8}$$

where

$$\begin{aligned}Q_1&=\pi^2Lc^{-2}\tilde{C}_n^2\kappa_0^{2-\alpha}A(\alpha)U\left(1;2-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_m^2}\right)\\&\cong\pi^2Lc^{-2}\tilde{C}_n^2\kappa_0^{2-\alpha}A(\alpha)\left[\frac{\Gamma(\alpha/2-1)}{\Gamma(\alpha/2)}+\Gamma(1-\alpha/2)\left(\frac{\kappa_0}{\kappa_m}\right)^{\alpha-2}\right].\end{aligned} \tag{9}$$

U(·) in Eq. (9) denotes a confluent hypergeometric function of the second kind. In arriving at the second step of Eq. (9), we have assumed $(\kappa_0/\kappa_m)^2\ll 1$. If the assumption of $l_0\to 0$, viz., $\kappa_m\to\infty$, is made, one can further obtain $Q_1=\pi^2Lc^{-2}\tilde{C}_n^2\kappa_0^{2-\alpha}A(\alpha)\Gamma(\alpha/2-1)/\Gamma(\alpha/2)$, which is consistent with Eq. (31) in Chapter 18 of Ref. [7] when $\alpha=11/3$.

Under the near-field condition, following Young et al. [12], using Eqs. (4) and (8) to obtain $\Gamma_2(\cdot)$ and evaluating the double integral in Eq. (2), one finds

$$\langle I(\mathbf{r},L;t)\rangle\cong a_0^2\frac{T_0}{T_1}\exp\left(-\frac{2r^2}{W_0^2}\right)\exp\left[-\frac{2(t-L/c)^2}{T_1^2}\right], \tag{10}$$

where $T_1=\sqrt{T_0^2+8Q_1}$. Based on Eq. (10), one can deduce that the mean arrival time t_m of the pulse under the near-field condition equals L/c. On the other hand, under the far-field condition together with the narrowband approximation, following Andrews and Phillips [7], in a similar manner, yields

$$\langle I(\mathbf{r},L;t)\rangle\cong a_0^2\left(\frac{W_0^2}{2Lc}\right)^2\frac{T_0^2[T_0^2(1+\omega_0^2T_0^2)+W_0^2r^2/(Lc)^2]}{T_1[T_0^2+W_0^2r^2/(Lc)^2]^{5/2}}$$