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On-axis Gaussian beam scattering by a spheroid with a rotationally uniaxial anisotropic spherical inclusion

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ABSTRACT

Based on the generalized Lorenz–Mie theory (GLMT) framework, a theoretical procedure to determine the scattered fields of a spheroid with a rotationally uniaxial anisotropic spherical inclusion, for incidence of an on-axis Gaussian beam described by a localized beam model, is presented. Numerical results of the normalized differential scattering cross section are presented, and the scattering characteristics are discussed concisely.

1. Introduction

Currently, there has been great progress in the theoretical study of the scattering of electromagnetic waves by homogeneous and inhomogeneous isotropic particles, to quote a few of them in [1–6]. Analytical solutions to the electromagnetic plane wave scattering by uniaxial anisotropic spheres have also been presented by some researchers [7–9]. Knowledge of the scattered field is required in many areas, such as in investigations of microwave scattering by raindrops and water-coated hailstones, and of light scattering by small chemical and biological particles. In view of the possibility of a dielectric particle having an anisotropic nucleus, it will be very interesting to analyze the theoretical model of the shaped beam scattering by a dielectric isotropic particle with an anisotropic inclusion. In this paper, we consider the on-axis Gaussian beam scattering by a spheroid with a rotationally uniaxial anisotropic spherical inclusion.

The paper is organized as follows. In Section 2, a theoretical procedure is given for determining the scattered fields of an on-axis Gaussian beam by a spheroid with a rotationally uniaxial anisotropic spherical inclusion. Section 3 provides the numerical results of on-axis Gaussian beam scattering properties. Section 4 is a conclusion.

2. Formulation

2.1. Expansions of the on-axis Gaussian beam, scattered fields as well as the fields within the spheroid in spheroidal coordinates

As shown in Fig. 1a Gaussian beam propagates in free space and from the negative z' to the positive z' axis of the Cartesian coordinate system $O'x'y'z'$, with the middle of its beam waist located at origin O'. The system $Ox''y''z''$ is parallel to $O'x'y'z'$, and the Cartesian coordinates of origin O in $O'x'y'z'$ are $(x_0=y_0=0,z_0)$ (on-axis case). The system $Oxyz$ is obtained by a rigid-body rotation of $Ox''y''z''$ through Euler angles α and β [10], and a spheroid with a rotationally uniaxial anisotropic spherical inclusion at the center is natural to $Oxyz$. The semifocal distance, semimajor and semiminor axes are denoted by f, a and b for the outer surface of the spheroid, and the radius by r_1 for the spherical inclusion. In this paper, the time-dependent part of the electromagnetic fields is assumed to be $\exp(-i\omega t)$.

As described in [11–14], the electromagnetic fields of the incident Gaussian beam, for the TM mode (TM or TE mode depending on whether the plane wave contribution of the magnetic or electric vector of the Gaussian beam vibrates perpendicularly to the incidence plane, i.e., the plane defined by the direction of propagation of the incident Gaussian beam and the z axis) [14], can be expanded in terms of the spheroidal vector wave functions $\begin{bmatrix} M & N \end{bmatrix}^{r(1)}_{\substack{e\\o}mn}(c,\zeta,\eta,\phi)$ with respect to $Oxyz$, as follows:

$$E^i = E_0 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} i^n \Big[G^m_{n,TE} M^{r(1)}_{emn}(c,\zeta,\eta,\phi)$$

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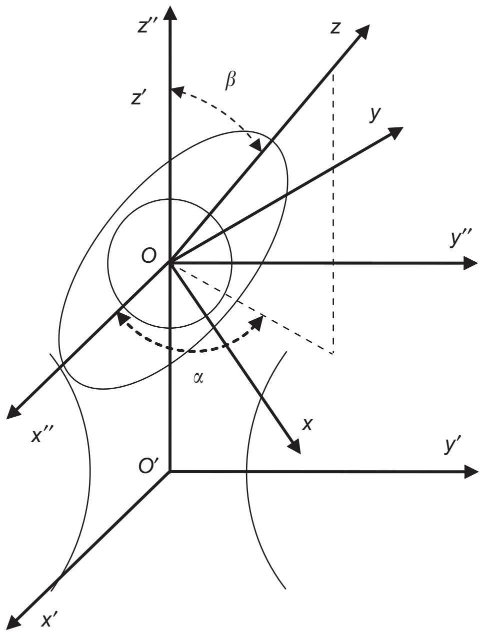

Fig. 1. The cartesian coordinate system $Ox''y''z''$ is parallel to the Gaussian beam coordinate system $O'x'y'z'$, and the cartesian coordinates of origin O in $O'x'y'z'$ are ($x_0=y_0=0,z_0$).$Oxyz$ is obtained by a rigid-body rotation of $Ox''y''z''$ through Euler angles α and β. A spheroid with a rotationally uniaxial anisotropic spherical inclusion is natural to $Oxyz$.

$$+G'^{m}_{n,TE}M^{r(1)}_{omn}(c,\zeta,\eta,\phi)-iG'^{m}_{n,TM}N^{r(1)}_{emn}(c,\zeta,\eta,\phi)$$

$$+iG^{m}_{n,TM}N^{r(1)}_{omn}(c,\zeta,\eta,\phi)\Big] \quad (1)$$

where $c=kf$, and the expansion coefficients or beam shape coefficients $G^m_{n,\mathrm{TE}}$, $G'^m_{n,\mathrm{TE}}$, $G^m_{n,\mathrm{TM}}$ and $G'^m_{n,\mathrm{TM}}$ can be expressed explicitly as [11,15,16]

$$\begin{bmatrix} G^m_{n,\mathrm{TE}} \\ G'^m_{n,\mathrm{TE}} \\ G^m_{n,\mathrm{TM}} \\ G'^m_{n,\mathrm{TM}} \end{bmatrix} = \frac{2(-1)^{m-1}}{N_{mn}}\sum_{r=2,1}^{\infty}{}' \frac{d_r^{mn}(c)}{(r+m)(r+m+1)}$$

$$\times g_{r+m}\begin{bmatrix} (2-\delta_{m0})\frac{dP^m_{r+m}(\cos\beta)}{d\beta}\sin\alpha \\ 2m\frac{P^m_{r+m}(\cos\beta)}{\sin\beta}\cos\alpha \\ 2m\frac{P^m_{r+m}(\cos\beta)}{\sin\beta}\sin\alpha \\ (2-\delta_{m0})\frac{dP^m_{r+m}(\cos\beta)}{d\beta}\cos\alpha \end{bmatrix} \quad (2)$$

where the prime over the summation sign indicates that the summation is over even values of r when $n-m$ is even and over odd values of r when $n-m$ is odd, and $d_q^{mn}(c)$ are the spheroidal expansion coefficients [17], and $\delta_{m0}=0$ when $m\neq 0$, and $\delta_{00}=1$.

The g_{r+m} coefficients in (2), when the Davis–Barton model of the Gaussian beam is used [18], can be computed by using the localized approximation as [19–21]

$$g_{r+m}=\frac{\exp(ikz_0)}{1+2isz_0/w_0}\exp\left[\frac{-s^2(r+m+1/2)^2}{1+2isz_0/w_0}\right] \quad (3)$$

where $s=\frac{1}{kw_0}$, and w_0 is the beam waist radius.

For the TE mode of the incident Gaussian beam, the corresponding expansions can be obtained only by replacing $G^m_{n,\mathrm{TE}}$ in (1) by $-G'^m_{n,\mathrm{TM}}$, $G'^m_{n,\mathrm{TE}}$ by $G^m_{n,\mathrm{TM}}$, $G'^m_{n,\mathrm{TM}}$ by $G^m_{n,\mathrm{TE}}$, and $G^m_{n,\mathrm{TM}}$ by $-G'^m_{n,\mathrm{TE}}$.

The scattered fields as well as the fields within the spheroid can correspondingly be expanded as follows:

$$E^s=E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}i^n\Big[\beta_{mn}M^{r(3)}_{emn}(c,\zeta,\eta,\phi)+\beta'_{mn}M^{r(3)}_{omn}(c,\zeta,\eta,\phi)$$
$$-i\alpha'_{mn}N^{r(3)}_{emn}(c,\zeta,\eta,\phi)+i\alpha_{mn}N^{r(3)}_{omn}(c,\zeta,\eta,\phi)\Big] \quad (4)$$

$$E^w=E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}i^n\Big[\delta_{mn}M^{r(1)}_{emn}(c',\zeta,\eta,\phi)+\delta'_{mn}M^{r(3)}_{emn}(c',\zeta,\eta,\phi)$$
$$+\chi_{mn}M^{r(1)}_{omn}(c',\zeta,\eta,\phi)+\chi'_{mn}M^{r(3)}_{omn}(c',\zeta,\eta,\phi)-i\tau_{mn}N^{r(1)}_{emn}(c',\zeta,\eta,\phi)$$
$$-i\tau'_{mn}N^{r(3)}_{emn}(c',\zeta,\eta,\phi)+i\gamma_{mn}N^{r(1)}_{omn}(c',\zeta,\eta,\phi)+i\gamma'_{mn}N^{r(3)}_{omn}(c',\zeta,\eta,\phi)\Big] \quad (5)$$

where $c'=fk'$, $k'=k\tilde{n}$, and $\tilde{n}$ is the refractive index of the material of the spheroid relative to that of free space.

For the sake of brevity, only the electric fields are written, and the corresponding expansions of the magnetic fields can be obtained with the following relations

$$H=\frac{1}{i\omega\mu}\nabla\times E,\quad M_{\substack{e\\o}mn}=\frac{1}{k}\nabla\times N_{\substack{e\\o}mn},\quad N_{\substack{e\\o}mn}=\frac{1}{k}\nabla\times M_{\substack{e\\o}mn} \quad (6)$$

2.2. *Description of the electromagnetic fields within the rotationally uniaxial anisotropic spherical inclusion*

In a rotationally uniaxial anisotropic medium, source-free Maxwell equations are written as

$$\nabla\times E=i\omega B,\quad \nabla\times H=-i\omega D \quad (7)$$

and the constitutive relations for the medium as

$$D=\bar{\bar{\varepsilon}}\times E,\quad B=\bar{\bar{\mu}}\times H \quad (8)$$

where the permittivity and permeability tensors $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$ are characterized by

$$\bar{\bar{\varepsilon}}=(\varepsilon_r\hat{r}\hat{r}+\varepsilon_t\hat{\theta}\hat{\theta}+\varepsilon_t\hat{\phi}\hat{\phi}),\quad \bar{\bar{\mu}}=(\mu_r\hat{r}\hat{r}+\mu_t\hat{\theta}\hat{\theta}+\mu_t\hat{\phi}\hat{\phi}) \quad (9)$$

The electromagnetic fields within the rotationally uniaxial anisotropic medium can be decomposed into the TE and TM modes (with respect to $\hat{r}$), by introducing the scalar potentials Φ_{TE} and Φ_{TM} [7,8]

$$D=-\nabla\times(\hat{r}\Phi_{\mathrm{TE}}) \quad (10)$$

$$B=\nabla\times(\hat{r}\Phi_{\mathrm{TM}}) \quad (11)$$

Combining (7), (8) and (10), after some algebra we can have

$$\omega^2\nabla\Phi_{\mathrm{TE}}\times\hat{r}=\nabla\times\left\{\bar{\bar{\mu}}^{-1}\times\nabla\times\left[\bar{\bar{\varepsilon}}^{-1}\times(\nabla\Phi_{\mathrm{TE}}\times\hat{r})\right]\right\} \quad (12)$$

From (12) the wave equation for Φ_{TE} can be written in spherical coordinates as

$$\frac{1}{R_{\mathrm{TE}}}\frac{\partial^2\Phi_{\mathrm{TE}}}{\partial r^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\Phi_{\mathrm{TE}}}{\partial\theta})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi_{\mathrm{TE}}}{\partial\phi^2}+\frac{1}{R_{\mathrm{TE}}}k_t^2\Phi_{\mathrm{TE}}=0 \quad (13)$$

where $R_{\mathrm{TE}}=\mu_t/\mu_r$, $k_t^2=\omega^2\varepsilon_t\mu_t$.The solution to (13) can be obtained by means of the method of separation of variables, which is of the form

$$\Phi_{\substack{e\\o}\mathrm{TE}}=rj_{\nu_{\mathrm{TE}(n)}}(k_tr)P_n^m(\cos\theta)\begin{matrix}\cos m\phi\\ \sin m\phi\end{matrix} \quad (14)$$

where $\nu_{\mathrm{TE}}(n)=\frac{-1+\sqrt{1+4R_{\mathrm{TE}}n(n+1)}}{2}$, and $j_{\nu_{\mathrm{TE}(n)}}(k_tr)$ is the spherical Bessel function of order $\nu_{\mathrm{TE}}(n)$.From (7), (8) and (11) the wave equation for $\Phi_{\substack{e\\o}\mathrm{TM}}$, and then its solution can also be obtained, which have the same forms as (13) and (14), except that in them