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A moment method analysis of the gain spectrum in bi-directionally pumped Raman amplifiers through continuous-spectrum radiation

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ABSTRACT

A semi-analytical method is proposed to solve the equations governing a bi-directionally pumped Raman amplifier through continuous-spectrum radiation. The governing equations are systems of uncountable Nonlinear Ordinary Differential Equation (NODE). By applying the moment method, the uncountable system of NODE is reduced to a system of finite number of NODEs. This system of equations is solved numerically and the results are compared with that of the full numerical method. It was shown that the moment method is a powerful and efficient technique for the analysis of a bi-directionally pumped Raman amplifier through continuous-spectrum radiation.

1. Introduction

The strong interest in using Raman amplification results from the fact that the system performance may be improved; that the gain at any signal wavelength may be obtained simply by choosing the proper pump wavelength, and just as importantly, that gain spectrum may be broadened by combining multiple pump sources at different wavelengths [1]. However, due to the complexity and cost, the use of multiple pump wavelengths may be less attractive from a commercial point of view.

Recently a fiber Raman amplifier (FRA) using incoherent pumping has gained interest and has been experimentally demonstrated. In comparison with coherent pumping, incoherent pumping has the advantages of polarization insensitivity and the reduction of nonlinear effects, such as the stimulated Brillion scattering (SBS) of pumps and four-wave mixing (FWM) of pump–pump, pump–signal, and pump–noise [2].

It is shown that broadband pumping can broaden the Raman gain curve. The Raman gain profile produced by a pump is the convolution of the pump spectrum and the Raman gain curve. If a broadband pump source is used then by properly shaping the pump spectrum a flatter Raman gain profile can be obtained. This can reduce the number of pump diodes needed in a system [1,3–5].

A set of broadband pump sources are needed to produce a wideband pump for the fiber Raman amplifier. The required spectral broadening of emission lines is achieved by using diode lasers without spectrally selective elements like narrowband mirrors or Brag grating. The emission bandwidths of diode lasers in the 14XXnm region without spectrally selective element can be in the range 10–15 nm [6–8]. Other methods, such as incoherent pumping [9,10] and highly non-linear fiber (HNLF) broadening [7,8,11], can also be applied to produce broadband pump sources.

It is essential to find a semi-analytical method to solve the equations governing continuous-spectrum pumped Raman amplifiers. The governing equation of an optical Raman amplifier with a continuous-spectrum pump is a two-point boundary value problem with uncountable-coupled nonlinear differential equations, and the moment method is used to reduce it into a set of countable coupled nonlinear differential equations.

In our previous work [12], the moment method was employed for one directional pumped FRA to reduce the system of uncountable governing equations into a set of countable coupled nonlinear differential equations. Here the moment method is adopted to solve the governing equations of the bi-directionally pumped Raman amplifiers with continuous-spectrum pump. Comparing the results with that of the full numerical method [13], it is shown that the proposed method is a powerful method for solving bi-directionally pumped Raman amplifiers.

2. Theoretical method

Wave propagation in the multi-pump fiber Raman amplifier is characterized by a variety of physical effects, the major influences of which, for the gain profile design of bi-directionally pumped Raman amplifiers, are pump-to-pump and pump-to-signal stimulated Raman scattering, as well as fiber loss experienced by both

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pump and signal waves. In the steady state, the coupled equations can be described as [14]:

$$\frac{dS(v_k,\, z)}{dz} = S(v_k,\, z)\ \Big[\alpha(v_k) + \sum_{\hat{k}=1}^{n_s} g(v_k,\ v_{\hat{k}})S(v_{\hat{k}},\ z) + \int_0^{+\infty} g(v_k,\ \hat{v})[P^{+}(\hat{v},\ z) + P(\hat{v},\ z)]\, d\hat{v}\Big]$$
$$k = 1, 2,\ \ldots,\ n_s \tag{1}$$

$$\frac{dP^{\pm}(\hat{v},\, z)}{dz} = \pm P^{\pm}(\hat{v},\ z)\ \Big[\alpha(\hat{v}) + \sum_{\hat{k}=1}^{n_s} g(\hat{v},\ v_{\hat{k}})S(v_{\hat{k}},\ z) + \int_0^{+\infty} g(\hat{v},\ v)[P^{+}(v,\ z) + P(v,\ z)]\, dv\Big]$$
$$\hat{v} \in R \tag{2}$$

where z is the distance along the fiber, $S(v_k, z)$ is the signal power at frequency v_k, $P^{+}(\hat{v}, z)$ and $P^{-}(\hat{v}, z)$ are pump power spectrum of forward- and backward-propagating waves at frequency $\hat{v}$ at point z, respectively. In Eq. (2), one uses the + sign for forward-propagating waves and – sign for backward-propagating waves. Here n_s is the number of signal channels and $\alpha(v)$ is the attenuation coefficient for a wave with frequency v. The gain coefficient $g(v, v')$ describes the energy transferred by stimulated Raman scattering (in stokes direction) between v and v' waves and is given by [12]:

$$g(v,\ v') = \begin{cases} \dfrac{v'}{v_0}\,\dfrac{g_R(v'-v)}{\Gamma A_{eff}} & v < v' \\ 0 & v = v' \\ -\frac{v^2}{v' v_0}\,\dfrac{g_R(v-v')}{\Gamma A_{eff}} & v > v' \end{cases} \tag{3}$$

where $g_R(\Delta v)$ is the Raman gain coefficient measured at reference pump frequency v_0, A_{eff} the effective area of optical fiber and the factor of Γ accounts for polarization randomization effects, whose value lies between 1 and 2. In the following calculations it is assumed that $A_{eff} = 80 \times 10^{-12}\,\mathrm{m}^2$, $v_0 = 300\,\mathrm{THz}$ and $\Gamma = 2$.

On the right hand side of Eqs. (1) and (2), the first terms are due to the fiber loss, the second terms are due to the Raman signal–signal and pump–signal interactions and the third terms correspond to the Raman signal–pump and pump–pump interactions in the fiber amplifier, respectively. Since noise has negligible effect on the gain of the Raman amplifier, in this analysis the effect of spontaneous Raman scattering and Rayleigh backscattering are neglected [14].

Since the number of equations in Eq. (2) is as many as the number of real numbers, the system of governing equations is a system of uncountable-coupled nonlinear differential equations. The moment method is used so as to reduce the system of governing equations into a countable system of equations. The moments are defined by the set:

$$q(\pm,\ m,\ n,\ t,\ \ell) = \{q^{(\pm,\ m,\ n,\ t)}_{k_1, k_2,\ \ldots,\ k_\ell}(z) | k_1,\ k_2,\ \ldots,\ k_\ell = 1,\ 2,\ \ldots,\ n_s\}$$
$$m = 0,\ 1,\ 2,\ \ldots \quad n = 0,\ 1,\ 2,\ \ldots \quad t = 0,\ 1,\ 2,\ \ldots \quad \ell = 1,\ 2,\ 3,. \tag{4}$$

With the following elements:

$$q^{(\pm,\ m,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(z) = \int_0^{+\infty} \alpha^m(\hat{v})Q_+^n(\hat{v},\ z)Q^t(\hat{v},\ z)P^{\pm}(\hat{v},\ z) \times g(v_{k_1},\ \hat{v})g(\hat{v},\ v_{k_2}) \ldots g(\hat{v},\ v_{k_\ell})\, d\hat{v} \tag{5}$$

which are called moment functions and $Q_{\pm}(\hat{v}, z)$ is defined by the following relation:

$$Q_{\pm}(\hat{v},\ z) = \int_0^{+\infty} g(\hat{v},\ v)P^{\pm}(\hat{v},\ z)\, dv \tag{6}$$

The effect of pump–pump interactions is presented in the $Q_{\pm}(\hat{v}, z)$ variable, which is defined in Eq. (6) and is appeared in moment functions. The moment functions are symmetric with respect to all lower indices except for the first one. The governing equations (Eqs. (1) and (2)) are rewritten with respect to the moment functions:

$$\frac{dS(v_k,\, z)}{dz} = S(v_k,\ z)\ \Big[\alpha(v_k) + q_k^{(+,\ 0,\ 0,\ 0)}(z) + q_k^{(,\ 0,\ 0,\ 0)}(z) + \sum_{\hat{k}=1}^{n_s} g(v_k,\ v_{\hat{k}})S(v_{\hat{k}},\ z)\Big] \tag{7}$$

$$S(v_k,\ 0) = S_{k0}, \quad k = 1,\ 2,\ \ldots,\ n_s \tag{8}$$

$$\frac{dP^{\pm}(\hat{v},\, z)}{dz} = \pm P^{\pm}(\hat{v},\ z)\ \Big[\alpha(\hat{v}) + Q_+(\hat{v},\ z) + Q(\hat{v},\ z) + \sum_{\hat{k}=1}^{n_s} g(\hat{v},\ v_{\hat{k}})S(v_{\hat{k}},\ z)\Big] \tag{9}$$

$$P^{+}(\hat{v},\ 0) = P^{+}(\hat{v}), \quad P(\hat{v},\ L) = P(\hat{v}), \quad \hat{v} \in R \tag{10}$$

To obtain the differential equations of the moment elements, both sides of Eq. (9) are multiplied by $\alpha^m(\hat{v})Q_+^n(\hat{v})Q_-^t(\hat{v})g(v_{k_1},\ \hat{v})$ $g(\hat{v},\ v_{k_2}) \ldots g(\hat{v},\ v_{k_\ell})$ and integrated over all frequencies:

$$\frac{dq^{(\pm,\ m,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(z)}{dz} = \pm\Big[q^{(\pm,\ m+1,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(z) + q^{(\pm,\ m,\ n+1,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(z) + q^{(\pm,\ m,\ n,\ t+1)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(z) + \sum_{\hat{k}=1}^{n_s} S(v_{\hat{k}},\ z)q^{(\pm,\ m,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell \hat{k}}(z)\Big] \tag{11}$$

For a bi-directionally pumped Raman amplifier with a fiber span length of L, the boundary conditions are defined at $(z = 0)$ for signal waves $S(v_k, 0) = S_{k0}(k = 1, 2, \ldots, n_s)$ and forward pump wave $P^{+}(\hat{v},\ 0) = P^{+}(\hat{v})$ and at $(z = L)$ for backward pump wave $P^{-}(\hat{v},\ L) = P^{-}(\hat{v})$, so the boundary conditions for moment elements are obtained by the following relations:

$$q^{(+,\ m,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(0) = \int_0^{+\infty} \alpha^m(\hat{v})Q_+^n(\hat{v},\ L)Q^t(\hat{v},\ L)P^{+}(\hat{v})g(k_1,\ \hat{v}) \ldots g(\hat{v},\ k_\ell)\, d\hat{v}\, q^{(,\ m,\ n,\ t)}_{k_1,\ k_2,\ \ldots,\ k_\ell}(L)$$
$$= \int_0^{+\infty} \alpha^m(\hat{v})Q_+^n(\hat{v},\ L)Q^t(\hat{v},\ L)P(\hat{v})g(k_1,\ \hat{v}) \ldots g(\hat{v},\ k_\ell)\, d\hat{v} \tag{12}$$

Since the $|g(\hat{v},\ v_k)|$, $|Q_{\pm}(\hat{v})|$ and $\alpha(v)$ are less than unity [15] ($|g(\hat{v},\ v_k)| \approx 10^{-4}(1/\mathrm{mW\,km})$ and $\alpha(v) \approx 10^{1}(\mathrm{dB/km})$, the absolute value of moment elements leads to a set with zero elements. But the rate of convergence with respect to the number of $|g(\hat{v},\ v_k)|$ and powers of $|Q_{\pm}(\hat{v})|$ in moments is faster than powers of $\alpha(v)$, because $|g(\hat{v},\ v_k)|$ and $|Q_{\pm}(\hat{v})|$ have the same order of magnitude and their magnitude are less than $\alpha(v)$, so we have to arrange the moment's order as follows:

$$q(M,\ N) = \{q(\pm,\ m,\ n,\ t,\ \ell) : M = m,\ \ b = n + t + \ell\} \tag{13}$$

Hence, the countable system of governing equations (Eqs. (7) and (11)) can be truncated to a finite moment order (M, b), which can be obtained by the upper bound of the solution error. The number of signals n_s and the moment's order (M, b) determine the