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Explicit finite difference solution of the power flow equation in W-type optical fibers

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ABSTRACT

Using the power flow equation, we have calculated spatial transients of power distribution and a steady-state distribution that are due to coupling of guided to leaky modes in W-type optical fibers (doubly clad fibers). A numerical solution has been obtained by the explicit finite difference method. Results show that power distribution in W-type optical fibers depends on both the intermediate layer width and the coupling strength. W-shaped index profile of optical fibers is effective in reducing modal dispersion and therefore in improving the fiber bandwidth. We have also shown that explicit finite difference method is effective and accurate for solving the power flow equation in W-type optical fibers.

1. Introduction

There has been a large increase in the demand for transmission capacity of communication links. Singly clad (SC) glass optical fibers have been the preferred transmission medium in high-capacity communications networks and long-distance communications systems. In contrast, SC plastic optical fibers (POFs) are usually considered for short data links. POF systems show limitations in the achievable transmission rate due to their limited bandwidth and high attenuation. Numerous efforts and solutions to overcome those limitations have been proposed, ranging from spatial modulation [1] and detection techniques [2], equalization [3], modal dispersion compensation [4], and restricted modal launc. In spite of variety of efforts to improve multimode fiber based systems, only limited work has been done recently in multimode-fiber profile design and analysis. For the reasons described above, there is a need for practical multimode fiber design which could result in the improvement of the fiber transmission characteristics, especially fiber bandwidth. One of the promising choices is to employ W-type fibers. Recently, it has been experimentally demonstrated that POFs with the W-shaped index profile enabled fiber bandwidths in excess of those associated with graded index POFs [5].

The W-type optical fiber has a wider transmission bandwidth and lower bending losses compared to those of a corresponding SC optical fiber. This is due to the fact that the number of guided modes in W-type fiber is reduced because the intermediate layer decreases the effective numerical aperture of the fiber and hence the number of guided modes, and also because the guided modes are tightly confined within the core region [6]. A typical bandwidth-distance product for glass optical fibers is ~30 MHz km for SC and ~50 MHz km for W-type, whereas the corresponding figures for POFs are ~15 MHz km (SC) and ~200 MHz km (W-type) [5,7–9].

Transmission characteristics of multimode optical fibers depend strongly upon the differential mode-attenuation and rate of mode coupling. Modal attenuation is the result of light absorption and scattering in the fiber material. Mode coupling is diffraction of light that transfers power from one mode to another due to random anomalies in multimode optical fibers (microscopic bends, voids and cracks, diameter variation and density fluctuation). Due to mode coupling, steady-state power distribution is achieved at some distance z_s from the input fiber end whereby a unique normalized intensity distribution results across the far field disk pattern regardless of the mode(s) launched. Steady-state distribution indicates the complete independence of the output light distribution from launch conditions. Modal attenuation limits the power that can be transmitted along the fiber. Modal dispersion in optical fibers is reduced through mode coupling. This increases fiber bandwidth but, on the other hand, it also increases the amount of power radiated in fiber curves or bends [10], significantly changing the output-field properties and degrading the beam quality. W-type fibers, having an intermediate layer between the core and cladding, have somewhat different properties from those of SC fibers due to the existence of lossy leaky modes in the intermediate layer. Mode coupling from guided to lossy leaky modes in W-type fibers has not been fully analyzed yet. Many fiber

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junctions and static bends are expected in the W-type optical fiber networks, presenting an additional cause of mode coupling. Due to mode coupling, the optical energy of the low-order modes would be coupled to higher order modes even if only the low-order ones are launched selectively by the restricted launch condition. Since these higher order modes can degrade the bandwidth performance of the W-type fibers, the group delay difference among all the modes (from lowest to highest) should be minimized by the refractive index profile [11]. Because of the influence that modal attenuation, modal dispersion and mode coupling have on fiber transmission properties, it is necessary to have effective and accurate methods for calculating their rate in W-type optical fibers.

Output angular power distribution in the near and far fields of an optical fiber end has been studied extensively. By employing the power flow equation these patterns have been predicted as a function of the launch conditions and fiber length. The rate of mode coupling, which is the power transfer between modes, has been described by the "coupling coefficient" D [12–17].

In this work, using the power flow equation, we have investigated the influence of the width of the intermediate layer and the strength of mode coupling on spatial transients of power distribution as well as on the steady-state power distribution that are due to coupling of guided to leaky modes in W-type optical fibers.

2. Power flow equation

The time-independent power flow for multimode SC fibers is described by the following coupled-power Eq. [12]:

$$\frac{\partial P(\theta,z)}{\partial z} = -\alpha(\theta)P(\theta,z) + \frac{D}{\theta}\frac{\partial}{\partial\theta}\left(\theta\frac{\partial P(\theta,z)}{\partial\theta}\right) \tag{1}$$

This equation can be written in the following form:

$$\frac{\partial P(\theta,z)}{\partial z} = -\alpha(\theta)P(\theta,z) + \frac{D}{\theta}\frac{\partial P(\theta,z)}{\partial\theta} + D\frac{\partial^2 P(\theta,z)}{\partial\theta^2} \tag{2}$$

where $P(\theta,z)$ is the angular power distribution at distance z from the input end of the fiber, θ is the propagation angle with respect to the core axis, D is the coupling coefficient assumed constant [12,13] and $\alpha(\theta)=\alpha_0+\alpha_d(\theta)$ is the modal attenuation, where α_0 represents conventional losses (absorption and scattering). The term α_0 leads only to a multiplier $\exp(-\alpha_0 z)$ in the solution and is thus neglected. The boundary conditions are $P(\theta_m,z)=0$, where θ_m is the maximum propagation angle, and $D(\partial P/\partial\theta)=0$ at $\theta=0$.

Consider a W-type fiber with index profile shown in Fig. 1. The relative refractive index difference $\Delta_q=(n_0-n_q)/n_0$ between the core and intermediate layer is larger than the difference $\Delta_p=(n_0-n_p)/n_0$ between core and cladding, where n_0, n_q and n_p are refractive indices of the core, intermediate layer and cladding, respectively. In this structure, the modes whose propagation angles are between $\theta_p \cong (2\Delta_p)^{1/2}$ and $\theta_q \cong (2\Delta_q)^{1/2}$ are leaky modes [14]. Attenuation constants of leaky modes are given as [15]:

$$\alpha_L(\theta) = \frac{4(\theta^2-\theta_p^2)^{1/2}}{a(1-\theta^2)^{1/2}}\frac{\theta^2(\theta_q^2-\theta^2)}{\theta_q^2(\theta_q^2-\theta_p^2)}\exp\left[-2\delta a n_0 k_0(\theta_q^2-\theta^2)^{1/2}\right] \tag{3}$$

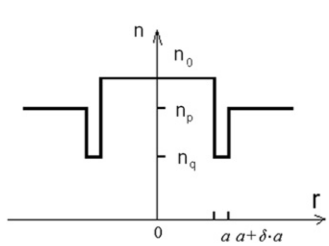

Fig. 1. Refractive index profile of a W-type fiber.

where k_0 is the free-space wave number, a is core radius and δa intermediate layer (inner cladding) width. In a SC fiber, experimental results show that attenuation remains constant throughout the guided-mode region and rises quite steeply in the radiation-mode region [16]. Consequently, the modal attenuation in a W-type fiber can be expressed as:

$$\alpha_d(\theta) = \begin{cases} 0 & \theta \le \theta_p \\ \alpha_L(\theta) & \theta_p < \theta < \theta_q \\ \infty & \theta \ge \theta_q \end{cases} \tag{4}$$

A W-type fiber can be regarded as a system consisting of SC_q fiber and cladding. In the SC_q fiber, modes having propagation angles smaller than the critical angle θ_q can be guided. When the SC_q fiber is coupled with surrounding medium of index n_p, the lower order modes, whose propagation angles are smaller than the critical angle of the SC_p fiber θ_p, remain guided. However, the higher order modes with angles between θ_p and θ_q are transformed into leaky modes. Because of the strong dependence of $\alpha_L(\theta)$ on the intermediate layer width δa, it is expected that the steady-state characteristics of a W-type fiber also depend on δa and coincide with those of SC_p and SC_q fibers in the limits of $\delta\to 0$ and $\delta\to\infty$, respectively. These two types of fiber have quite different features and need to be evaluated carefully so that conclusions could be applied in system design.

3. Numerical method

Since analytical solution of the power flow Eq. (2) with the attenuation constants of leaky modes in the form of (3) is not known, one has to solve Eq. (2) numerically. We now report, in our knowledge for the first time, the solution of the power flow Eq. (2) in W-type optical fiber using explicit difference method (EFDM). One should mention here that with respect to the stability and accuracy, it is more difficult to numerically solve Eq. (2) with modal attenuation given by Eq. (4) than it is for the case of a singly-clad fiber where $\alpha(\theta)\approx\alpha_0+A\theta^2$ [17].

In the 1970s and 1980s, implicit finite difference methods (IFDMs) were generally preferred over EFDMs. This trend has been changing with the advancement of computers, shifting the emphasis to EFDMs. Being often unconditionally stable, the IFDM allows larger step lengths. Nevertheless, this does not translate into IFDM's higher computational efficiency because extremely large matrices must be manipulated at each calculation step. We find that the EFDM is also simpler in addition to being computationally more efficient [14,17].

We used the central difference scheme to represent the $(\partial P(\theta,z))/\partial\theta$ and $(\partial^2 P(\theta,z))/\partial\theta^2$ terms, and the forward difference scheme for the derivative term $(\partial P(\theta,z))/\partial z$ [18]. Then, Eq. (2) reads:

$$P_{k,l+1} = \left(\frac{\Delta zD}{\Delta\theta^2} - \frac{\Delta zD}{2\theta_k\Delta\theta}\right)P_{k-1,l} + \left(1 - \frac{2\Delta zD}{\Delta\theta^2} - (\alpha_d)_k\Delta z\right)P_{k,l} + \left(\frac{\Delta zD}{2\theta_k\Delta\theta} + \frac{\Delta zD}{\Delta\theta^2}\right)P_{k+1,l} \tag{5}$$

where indexes k and l refer to the discretization step lengths $\Delta\theta$ and Δz for angle θ and length z, respectively, where:

$$(\alpha_d)_k = \begin{cases} 0 & \theta_k \le \theta_p \\ \frac{4(\theta_k^2-\theta_p^2)^{1/2}}{a(1-\theta_k^2)^{1/2}}\frac{\theta_k^2(\theta_q^2-\theta_k^2)}{\theta_q^2(\theta_q^2-\theta_p^2)}\exp\left[-2\delta a n_0 k_0(\theta_q^2-\theta_k^2)^{1/2}\right] & \theta_p < \theta_k < \theta_q \\ \infty & \theta_k \ge \theta_q \end{cases} \tag{6}$$

In the difference form, boundary conditions become $P_{N,l}=0$ and $P_{0,l}=P_{1,l}$, where $N=\theta_q/\Delta\theta$ is the grid dimension in θ direction.