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Self-calibration using two same circles

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ABSTRACT

Camera calibration is a fundamental step in 3D reconstruction and computer vision. Considering the easy accessibility of two same circles, a method for self-calibration based on two same circles is proposed. The proposed method does not need any prior knowledge and known camera parameters. By taking three photos of the object containing two same circles from different views, the close solution for intrinsic and extrinsic parameters are first obtained by the invariance of circular points and common tangent of two circles. Then the solution is refined by the nonlinear optimization. This method could get the intrinsic parameters as well as the extrinsic parameters without complicated matching process. Extensive results show the accuracy, robustness and wide applications of the proposed method.

1. Introduction

Camera calibration is a significant task in computer vision since the intrinsic parameters and extrinsic parameters of the camera are indispensable for 3D reconstruction. Some traditional calibration methods based on circular target are proposed due to its accuracy and robustness [1–3]. Compared with the traditional calibration methods, self-calibration is a flexible method. Until now, many results have been obtained on the self-calibration based on the circular target. Some researchers introduce the self-calibration methods based on concentric circles [4–7]. Meng et al. utilize a special temple which contains a circle and a bundle of lines crossing the center of the circle to obtain the intrinsic parameters [8]. Colombo et al. propose a new method based on coaxial circles [9]. Furthermore, some methods first obtain the image of infinity line by parallel lines or orthogonal lines and then get the image of circular points (ICPs) by solving the intersecting points of the infinity line and the ellipses transformed by the concentric circles. Finally, the camera parameters could be obtained by the ICPs [10–12]. Wu et al. propose two calibration methods based on two parallel circles and two coplanar circles [13,14]. The intrinsic parameters could be solved, but the solution for extrinsic parameters was not given. Chen et al. propose a calibration method from analytic geometry to solve both intrinsic parameters and extrinsic parameters [15]. But unfortunately, the equations derived by analytic geometry are too complicated and some intrinsic parameters are needed to be known beforehand. Gurdjos et al. analyze the Euclidean structure of parallel circles by ICPs, which makes a solid foundation for calibration based on parallel circles and coplanar circles [16]. Then Zheng et al. utilize ICPs and quadric enveloping lines to calibrate the camera [17]. This method could avoid the complicated equations derived by analytic geometry in Chen's method. However, for the projection of circle center the rotated matrix is needed in addition. Due to the perspective projection deviations [18], the true projection of circle center is not the fitting center of the ellipse projected by the circle. The solution for the true projection of circle center is not given. Furthermore, this method also needs some known intrinsic parameters.

The basic idea of all the above methods is the invariabilities of ICPs and absolute quadric curve. Due to the highly symmetric trait of the circle, these calibration methods could only obtain the intrinsic parameters. However, the extrinsic parameters are indispensable for 3D reconstruction. All the present methods could get simultaneously the intrinsic parameters and extrinsic parameters only under the condition that one part of intrinsic parameters are known or some other geometric objects (such as lines) are needed.

Due to tangent invariance in perspective projection, the common tangents of two circles have been used to obtain camera parameters. In this paper, the property of ICPs and the common tangents of two same circles are utilized to obtain intrinsic parameters and extrinsic parameters of the camera. The only requirement is that there are two same circles in the calibration template or geometric object. In the calibration the relative positions of the two same circles do not need to be known beforehand. Moreover, any prior information about the camera

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is not needed too. Because two same circles are quite easy to get and it could be extracted easily in practice (such as CD, pop can and bowl cover), therefore the proposed method could make the whole calibration process more flexible and simple.

2. Background

2.1. Camera parameters, absolute quadric curve and circular point

In projective transformation, a point in 3D world coordinate could be denoted as $M=[X,Y,Z]^T$ and the homogeneous coordinate of this point is $\tilde{M}=[X,Y,Z,t]^T$. When the point is on the infinite plane, t is equal to zero. Otherwise, t is equal to one. After projective projection, the corresponding point in 2D image coordinate is denoted as $m=[u,v]^T$ and the homogeneous coordinate is $\tilde{m}=[u,v,1]^T$. The relationship between the point in 3D world coordinate and its corresponding point in 2D image coordinate is

$$\lambda\tilde{m}=K[R,T]\tilde{M}. \tag{1}$$

where

$$K=\begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the intrinsic parameter matrix, R is the rotation matrix in the extrinsic parameters, T is the translation vector in the extrinsic parameters. In matrix K, s denotes the obliquity factor, f_x and f_y are the effective focal lengths in x axis and y axis, respectively, (u_0,v_0) is the principal point.

Absolute conic consists of points which are on the infinite plane. The point $[X,Y,Z]^T$ on the absolute quadric curve satisfies $X^2+Y^2+Z^2=0$, i.e., $\tilde{M}^T\tilde{M}=0$. According to Eq. (1), the image of absolute quadric curve should satisfy

$$\tilde{m}^TK^{-T}K^{-1}\tilde{m}=0. \tag{2}$$

Circular point is the intersection of line at infinity and the circle. There is one pair of circular points in a plane: $I(1,i,0,0)$, $J(1,-i,0,0)$ which satisfies

$$\begin{cases} x^2+y^2+z^2=0 \\ t=0 \end{cases}.$$

This illustrates that circular point is on the absolute quadric curve. If the ICPs m_I, m_J are known, the intrinsic matrix K could be obtained by Eq. (2).

2.2. Isotropic line and Laguerre theorem

Isotropic line is a virtual line which passes the circular point. Obviously, the intersection of isotropic line and line at infinity is the circular point. Let two common tangents be l_1, l_2, the slope of l_1, l_1 be λ_1, λ_2 and the angle between them be θ. Let isotropic line be m_1,m_2 and the slope of m_1,m_2 be $-i,+i$, respectively. μ is the temporary variable. Then we have

$$\mu=CR(l_1,l_2,m_1,m_2)=\frac{\sin(l_1,m_1)/\sin(l_2,m_1)}{\sin(l_1,m_2)/\sin(l_1,m_2)}$$
$$=\frac{(\lambda_1+i)(\lambda_2-i)}{(\lambda_2+i)(\lambda_1-i)}=\frac{(1+\lambda_1\lambda_2)+i(\lambda_2-\lambda_1)}{(1+\lambda_1\lambda_2)-i(\lambda_2-\lambda_1)}=\frac{1+i\frac{\lambda_2-\lambda_1}{1+\lambda_1\lambda_2}}{1-i\frac{\lambda_2-\lambda_1}{1+\lambda_1\lambda_2}}. \tag{3}$$

According to the straight-line angle formula, we obtain

$$\tan\theta=\frac{\lambda_2-\lambda_1}{1+\lambda_1\lambda_2}. \tag{4}$$

From simple algebraic knowledge, we have

$$\tan\theta=\frac{1}{i}\frac{e^{2i\theta}-1}{e^{2i\theta}+1}. \tag{5}$$

Substituting Eqs. (3) and (4) into Eq. (5), we obtain

$$\theta=\frac{1}{2i}\ln\mu. \tag{6}$$

This is the Laguerre Theorem [19]. According to this theorem and cross-ratio invariance in projective projection, if the ICPs and the images of two common intersecting lines are known, the actual angle between the two lines could be obtained.

3. Calibration procedures

The whole calibration procedures are shown in Fig. 1.

3.1. Compute the intrinsic parameters

From projective geometry, circular point is the intersecting point of line at infinity and the circle and the ICPs are the intersecting points of the image of line at infinity and image of the circle. For two coplanar circles, every circle could determine one pair of ICPs. Since there is only one pair of circular points on a plane, in the ideal case, ICPs determined by the two circles are the same and the ICPs are on the image of line at infinity. From the above analysis, it can be concluded that ICPs are included in the intersecting points of the images of two circles. Wu et al. have used this idea to obtain the ICPs [17]. Due to the fact that a standard circle is changed to an ellipse under the projective transformation, the ICPs are a pair of conjugate points.

Since two separate coplanar circles are used in this paper, the line which passes the circular points should not lie among the two circles. The line is shown as L_2 in Fig. 2(a). Therefore, we choose the method for determining circular points as follows. As shown in Fig. 2(a), in the world coordinate system, the intersecting points of two circles are two pairs of conjugate points. Line L_1 and line L_2 are determined by every pair of the conjugate points. O_1 and O_2 are the

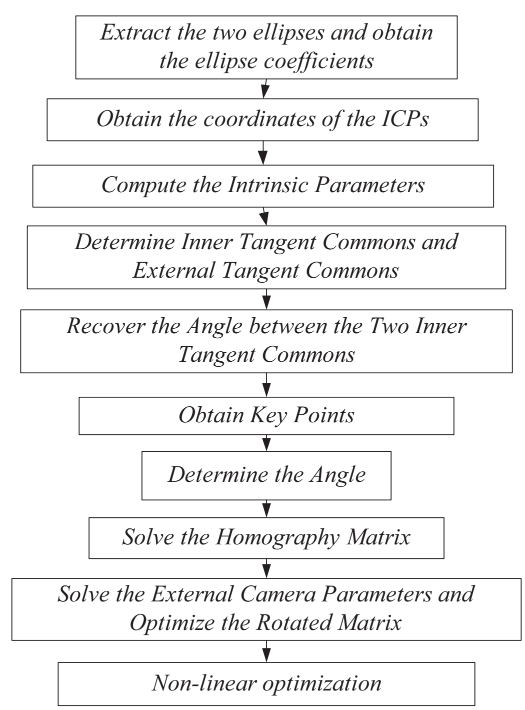

Fig. 1. Flow chart of the algorithm.