Optics & Laser Technology 43 (2011) 586–591

Contents lists available at ScienceDirect

Optics & Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Fiber-optic curvature sensor with optimized sensitive zone

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A R T I C L E   I N F O

Article history:
Received 23 March 2010
Received in revised form
2 August 2010
Accepted 17 August 2010

Keywords:
Geometrical optics
Curvature fiber-optic sensor
Orthogonal simulation experiments

A B S T R A C T

A novel fiber-optic sensor that can measure curvature directly has been developed previously. In this paper, the transduction of curvature to light intensity is described analytically by using the geometrical optics analysis. The mathematical model allows a quantitative optimization of the sensor without having to produce many sensors with slightly different combinations of parameters in order to accomplish a similar objective experimentally. The Monte Carlo simulation by ray tracing and an orthogonal matrix are used to optimize the fiber-optic sensor's configuration. The results show that the depth of the sensitive zone and the number of teeth are two main parameters that affect the sensor's sensitivity and the optimum number of teeth is 55, which is in agreement with the mathematical model.

1. Introduction

Although the term "deflection curvature" of loaded structures is very elementary in the field of strength of materials, this quantity largely had a theoretical significance only since it is difficult to measure in practice [1]. The material strain and deformation curvature are functionally related quantities and the latter quantity can usually be inferred from the former. The material strain can be measured by numerous devices. However, there are some disadvantages in traditional strain measurement. For example, the strain magnitude decreases with the decrease in structural thickness for a fixed amount of structural deformation. This implies that for any strain sensor, however sophisticated, a 'break-even thickness' exists so that the curvature sensor will provide higher sensitivity with structures thinner than that thickness [1–3].

In contrast, curvature measurement is position independent across a given structural cross-section. Even along the neutral plane where the strain is zero, the curvature of structural deformation can be measured [4]. However, sensors that can measure curvature are rare. An inexpensive conductive ink sensor can detect the curvature, but its measurement range is very narrow, from 0.01 to 0.1 mm^{-1} [5]. The wavelength-shift technique and the light interferometric technique can also be used to measure the bending deformation [6,7], but these systems are complex and require expensive spectrometers. Based on the principle that the light transmission loss will increase suddenly under large curvatures, a fiber-optic sensor was proposed to monitor respiratory chest circumference [8–10]. Because the surface of fiber is untreated, the sensitivity of the sensor is very low and it cannot distinguish the bending direction.

With the introduction of the curvature fiber-optic sensor reported recently [11,12], measurement of structural deformation curvature has become easier and more practicable. It is a fiber-optic, intensity-modulated curvature sensor. By different mechanical configuration of curvature fiber-optic sensor, many physical quantities, such as strain, torsion and position, can be calculated from the deflection-curvature measurements [12–14]. As is known, the sensitivity of untreated optical fibers is insufficient to detect the deformation of structures in bending. In order to increase the fiber's sensitivity to curvature, a sensitive zone is introduced on one side of the fiber by precision machining. The cutout produced removes a part of the fiber core and introduces a loss of light propagating along, as shown in Fig. 1 [1]. The sensor can distinguish positive bending (sensitive zone is on the convex side) from negative bending (sensitive zone is on the concave side) [15]. Positive bending decreases the throughput of light, and is distinguishable from negative bending, which increases the throughput.

The experimental optimization of curvature fiber-optic sensor with a diameter 0.25 mm was reported in [16]. In [15], a model of such sensor based on the classical reflection law was presented. In [1], the three-dimensional analysis of light propagation through the curvature fiber-optic sensor was described. In order to gain better qualitative understanding of this sensor, in this work a new

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doi:10.1016/j.optlastec.2010.08.005

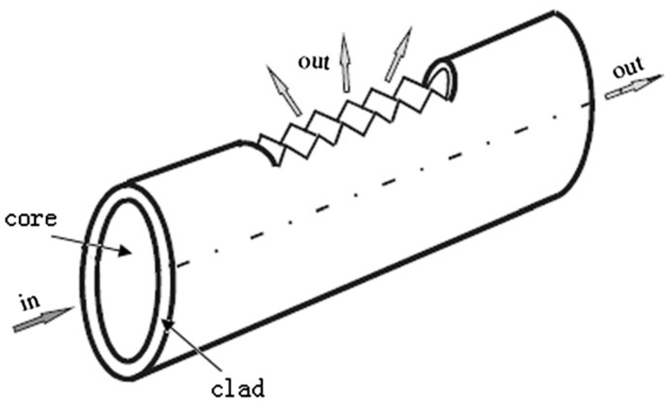

Fig. 1. A fiber with a sensitive zone [1].

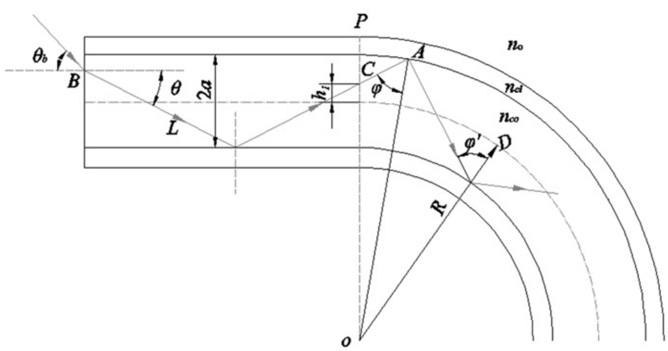

Fig. 2. The light propagation in a bent fiber.

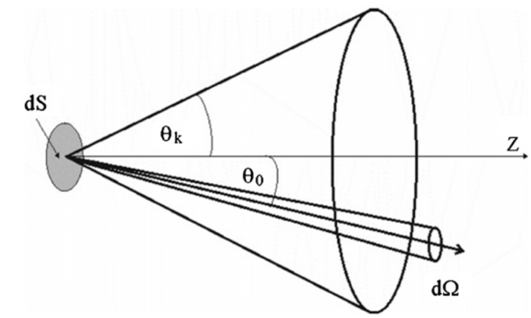

Fig. 3. Elemental source *dS* [15].

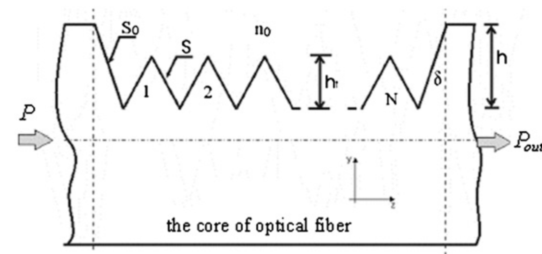

Fig. 4. Geometry of the sensitive zone [15].

mathematical model is proposed by using geometrical optics. The model provides the operation fundamentals and quantitative optimization of the sensor. In addition, we optimize the parameters of sensor with a diameter of 0.25 mm by ray-trace modeling using the orthogonal simulation experimental method.

2. The mathematical modeling

Fig. 2 shows a bent fiber. The radius of optical fiber is R, and the angle between beam of light L and the axis of fiber is θ. Beam of light L enters bent section at point C with an angle φ. The aperture angle θ_b of bent fiber is given by the following equation [17]:

$$\sin\theta_b = [n_{co}^2 - n_{cl}^2 (R+a)^2/(R+h_1)^2]^{1/2}/n_0 \tag{1}$$

where a is the radius of fiber core, n_{co} is the refractive index of fiber core, n_{cl} is the refractive index of fiber cladding, n_0 is the refractive index of transmission medium ($n_0=1$ to simplify calculations in this paper), h_1 is the distance between C and neutral plane of fiber. h_1 is a positive value when point C is at convex side of bent fiber, and negative value at concave side.

It is shown in (1) that the aperture angle increases along convex side on the cross-section of bent fiber. That is to say, the rays in the bent fiber are concentrated on the convex side.

As is shown in Fig. 3 [15], a light source (the light source involved in this paper is a Lambertian source, it can emit light in all directions) is modeled by the angular distribution of power from each of its elemental surface areas dS. It emits within a cone of half-angle θ_k. Assuming that the light intensity is $I(\theta_0)$ at angle θ_0 to the normal, the corresponding radiated power dP emits in the element of the solid angle $d\Omega$ is given by [15]

$$dP = I(\theta_0)d\Omega\, dS = I_0 \cos\theta_0\, r\, dr\, d\varphi \sin\theta_0\, d\theta_0\, d\theta_\varphi \tag{2}$$

where I_0 is a constant relative to the light source and is the intensity of the light normal to source; (r,φ) are polar coordinates relative to the fiber axis and $(\theta_0,\ \theta_\varphi)$ are spherical polar angles with respect to the incidence normal vector.

The total power P in straight fiber is obtained by integrating dP of (2) over the corresponding range of values of each of the four variables r, φ, θ_0 and θ_φ. Hence

$$P = I_0\pi^2 a^2 \sin^2\theta_c = I_0 \pi^2 a^2 n_{co}^2 (1 - n_{cl}^2/n_{co}^2). \tag{3}$$

Fig. 4 [15] shows the geometry of sensitive zone. To simplify calculations, the following assumptions are introduced:

(1) Teeth of the serrated sensitive zone are mutually identical regular prisms, as shown in Fig. 4.
(2) Surfaces of the teeth are smooth.
(3) The light leaking from the sensitive zone is absorbed absolutely by outside medium and cannot backtrack to the sensitive zone.

In Fig. 4, we start with a small element dS_0 of the surface S_0 and corresponding power dP_0 leaking from this segment in the space angle $d\Omega$. The coefficient of positive bending η_1 is introduced. From (2), dP_0 can be expressed as $dP_0 = \eta_1 I(\theta_0)\, dS_0\, d\Omega$. Under positive bending, ignoring the influence of the slope of surface S_0 on aperture angle, the light leakage power P_0 from surface S_0 is

$$\begin{aligned} P_0 &= \eta_1 \iint_{S_0} dS \int_0^{2\pi} d\theta_\varphi \int_0^{\theta_b} I_0 \cos\theta_0 \sin\theta_0\, d\theta_0 \\ &= 2\pi\eta_1 \int_{a-h}^{a} 2\cos\delta \sqrt{a^2 - h_1^2}\, dh_1 \int_0^{\theta_b} I_0 \cos\theta_0 \sin\theta_0\, d\theta_0 \\ &= 2\pi\eta_1 \cos\delta\, I_0 \int_{a-h}^{a} J(h_1)\, dh_1, \end{aligned} \tag{4}$$

where h is the depth of cuts, δ is the half angle of tooth as shown in Fig. 4 and

$$J(h_1) = \sqrt{a^2 - h_1^2}\,[n_{co}^2 - n_{cl}^2 (R+a)^2/(R+h_1)^2].$$