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Nonlinear interaction of intense hypergeometric Gaussian subfamily laser beams in plasma

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ABSTRACT

Propagation of Hypergeometric-Gaussian laser beam in a nonlinear plasma medium is investigated by considering the Source Dependent Expansion method. A subfamily of Hypergeometric-Gaussian beams with a non-negative, even and integer radial index, can be expressed as the linear superposition of finite number of Laguerre–Gaussian functions. Propagation of Hypergeometric-Gaussian beams in a nonlinear plasma medium depends on the value of radial index. The bright rings' number of these beams is changed during the propagation in plasma medium. The effect of beam vortex charge number l and initial (input) beam intensity on the self-focusing of Hypergeometric-Gaussian beams is explored. Also, by choosing the suitable initial conditions, Hypergeometric-Gaussian subfamily beams can be converted to one or more mode components that a typical of mode conversion may be occurred. The self-focusing of these winding beams can be used to control the focusing force and improve the electron bunch quality in laser plasma accelerators.

1. Introduction

Optical vortices have attracted a great deal of attention related to research and technological applications, such as particle acceleration, high harmonic generation and X-ray lasers [1–3]. A new family of the paraxial Helmholtz equation solution in free space, is hypergeometric (HyG) beam [4,5]. It is one of the optical vortices whose amplitude is proportional to the confluent hypergeometric function. A few studies have been done on HyG beam propagation. Recently, the propagation of HyG beams in a hyperbolic-index medium and a uniaxial crystal have been investigated [6–8]. HyG light beams can be generated with diffractive optical elements and computer-synthesized holograms [9,10]. Also, based on the HyG functions, generation of Hypergeometric Gaussian (HyGG) laser beams has been proposed [11]. HyGG beams can be generated with a liquid-crystal spatial light modulator [11]. Similar to the Laguerre–Gaussian (LG) beams, HyGG light beams have helical wavefront and their intensity profile consists of ring of light beam carrying orbital angular momentum [12]. So, they can be twisted like a corkscrew about the axis of propagation and have zero intensity at their center and hence they are also popularly named as twisted lights. The HyGG beam can be described by the phase singularity on axis with strength l that is called optical vortex charge number, and by the radial index p [12]. The laser beams propagation through the nonlinear plasma medium has been extensively investigated, whereas the self-focusing of laser beams has attracted most attention among many of the nonlinear phenomena [13–18]. Recently, several researches were conducted to study the propagation properties of Gaussian, Cosh-Gaussian, Hermit–Gaussian and Laguerre–Gaussian beams in plasma [19–21]. Nevertheless, the propagation of HyGG laser beam in nonlinear medium has been less studied. For this reason, the propagation of HyGG beams in the nonlinear plasma medium is surveyed. The propagation of laser beams in plasma can be numerically investigated using the Source Dependent Expansion (SDE) method [22,23]. In this research, the self-focusing of HyGG beams in nonlinear plasma medium under the paraxial approximation is surveyed. Here, the SDE method is applied to obtain equations which govern the beam width evolution of HyGG beam. As regards LG functions form an orthogonal and complete set, HyGG beam with a non-negative, even and integer radial number, can be expressed as a linear superposition of finite number of LG functions. By employing the SDE method in collisionless plasma, beam width evolution of this subfamily of beams is studied and equations related to the beam width evolution for each term of this expansion, considering ponderomotive force effect, are obtained. When an intense laser beam acts on collisionless plasma, ponderomotive force of the focused beam pushes the electrons out of high

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intensity region, reducing local electron density which leads to the further increase of plasma dielectric function and consequently, an even stronger self-focusing of laser beam occurs [24]. In this work, the beam width evolution of all members of expansion, coupled together, is investigated. In Section 2, all required relations to survey beams width evolution, based on the SDE method and employing ponderomotive force nonlinearity, will be considered. Section 3 is dedicated to numerical investigation of the beams width evolution. Finally, our conclusion is given in the last section.

2. Theoretical analysis

On the basis of paraxial approximation, which implies that the characteristic distance of intensity variation is much greater than wavelength, the amplitude of electric field, polarized along the x-axis, is denoted by A, satisfies the following equation:

$$\nabla_{\perp}^{2} A + 2ik\frac{\partial A}{\partial z} = -\frac{k^2 \Phi(AA^*)}{\epsilon_0'}A \tag{1}$$

where ϵ_0' and $\Phi(AA^*)$ are linear and nonlinear terms of permittivity, respectively, and k is the laser wavenumber. The star (*) is used to denote complex conjugate. The right hand side of Eq. (1) arises due to the nonlinear effects. In the free space or a medium without considering nonlinear effects, LG functions are the solution of Eq. (1) in the cylindrical coordinate system. Since, the LG functions form a complete-orthogonal set, one can expand an arbitrary laser beam versus superposition of LG basis as follows:

$$A = \sum_{q,l} d_{q,l} |LG\rangle_{q,l} \tag{2}$$

Here, $d_{q,l}$ is the coefficient of expansion. The LG functions for the slowly varying amplitude are considered as

$$|LG\rangle_{q,l} = a_{q,l}(z) D_q^l(r, z) e^{il\phi} \tag{3}$$

While

$$D_q^l(r, z) = \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_q^l\left(\frac{2r^2}{w(z)^2}\right) e^{-[1-i\beta(z)]r^2/w(z)^2} \tag{4a}$$

$$a_{q,l}(z) = \frac{E_0 w_0}{w(z)} q! \left(\frac{2}{\pi(l+q)!q!}\right)^{1/2} e^{i\psi_{q,l}(z)} \tag{4b}$$

In the above equations, $w(z)$ is the beam width, w_0 is the initial beam width, E_0 is the initial electric field amplitude, $\psi_{p,l}(z)$ is the Gouy phase, $\beta(z)$ is related to the curvature of wave front and $L_q^l()$ is the associated Laguerre polynomial. Substituting Eq. (2) in Eq. (1) and using the SDE method, the normalized beam width ($W = w(z)/w_0$) evolution of each term versus the normalized distance Z ($Z = z/z_R$, $z_R = kw_0^2/2$ is the Rayleigh length), in the absence of beam energy depletion, is as follows [21,22]:

$$\frac{d^2 W_i}{dZ^2} = \frac{1}{W_i^3} + \frac{kw_0^2}{W_i}\left(\frac{F_{q+1,l}(z)}{a_{q,l}(z)}\right)_i \tag{5a}$$

$$\left(\frac{F_{q+1,l}(z)}{a_{q,l}(z)}\right)_i = \frac{-k(q_i+1)!}{(l_i+q_i+1)!}\int_0^{\infty} \Phi(AA^*)\frac{A_{q_i,l_i}}{a_{q_i,l_i}(z)} D_{q_i+1}^{l_i^*}(\xi_i)d\xi_i \tag{5b}$$

where $\xi_i = 2r^2/w_i(z)^2$, i index refers to each LG function, and also, A and A_{q_i,l_i} are amplitude of electric field of an arbitrary laser beam and LG beam, respectively. In Eq. (5a), on the right hand side, the first term is vacuum diffraction and the second term is related to the nonlinear effects. In fact, by propagating a laser beam in a nonlinear medium, the $\Phi(AA^*)$ term plays an important role. According to Eq. (2), if LG laser beams have the same phase and intensity variation in medium, the laser beam amplitude A will be shape invariant. It is obvious from Eq. (5) by neglecting the nonlinear effects, the normalized beam width function is changed as $W = \sqrt{1+Z^2}$. In addition, Eqs. (2)–(4) show, for the sake of various phase of LG beams, the laser beam (superposition of LG functions) intensity is not shape invariant. Indeed, a superposition of LG beams with the same phase is remained shape invariant. Namely, Hermite–Gaussian (HG) beam can be expanded as a superposition of finite number of LG functions with the same phase [25] (algebraic details can be found in [25]); therefore, they are shape invariant beams. In another word, while HG beams propagate along the propagation distance, their intensity pattern does not change and is just scaled. Instead, the HyGG beams are not shape invariant even by propagating in the free space, as long as HyGG beam is propagated, one can see a dramatic change of intensity profile. Hereinafter, further important properties of this family will be investigated. If a collisionless plasma is chosen as the nonlinear medium, in Eq. (5b), $\Phi(AA^*)$ is given by [24]:

$$\Phi(AA^*) = \frac{\omega_p^2}{\omega^2}(1 - e^{-\eta AA^*}) \tag{6}$$

Here $\eta = \frac{e^2}{8m_e\omega^2 K_B T_{0e}}$; where e, m_e, K_B, T_{0e} and ω are electron charge, electron mass, Boltzmann constant, plasma equilibrium temperature and laser frequency, respectively. Also, ω is the laser frequency, $\omega_p = (\frac{e^2 n_e}{\epsilon_0 m_e})^{1/2}$ is the plasma frequency, n_e is the equilibrium electron density and ϵ_0 is the vacuum permittivity. When $\eta AA^* << 1$, one can rewritten the ponderomotive force nonlinearity as $\Phi(AA^*) \approx \frac{\omega_p^2}{\omega^2}AA^*$. In fact, when both the HyGG and LG beams are normalized, one can expand HyGG beams versus superposition of LG functions basis as [11]

$$|HyGG\rangle_{p,l} = B_{p,l}\frac{\Gamma(1+|l|+p/2)}{\Gamma(|l|+1)} i^{|l|+1} Z^{p/2}(Z+i)^{-[1+|l|+p/2]} \times e^{il\phi} R^{|l|} e^{-[iR^2/(Z+i)]} F_1\left(-p/2, |l|+1; \frac{R^2}{Z(Z+i)}\right) \tag{7a}$$

$$|HyGG\rangle_{p,l} = \sum_{q=0}^{\infty} C_{p,q} |LG\rangle_{q,l} \tag{7b}$$

$$C_{p,q} = \sqrt{\frac{(q+|l|)!}{\Gamma(p+|l|+1)!q!}}\frac{\Gamma(q-p/2)\Gamma(p/2+|l|+1)}{\Gamma(-p/2)\Gamma(q+|l|+1)} \tag{7c}$$

where $R = r/w_0$, $Z = z/z_R$, $\Gamma(\cdot)$ is the gamma function and $F_1(a, b; x)$ is a confluent hypergeometric function. If the non-negative, even and integer radial index p is chosen, the HyGG beams can be expressed as the linear superposition of a finite number of LG functions with the same vortex charge number l. In this case, expansion of $LG_{q,l}$ functions is confined to $0 \le q \le p/2$ and Eq. (7c) can be rewritten as [11]

$$C_{p,q} = (-1)^q \frac{(p/2)!(p/2+|l|)!}{(p/2-q)!\sqrt{q!(p+|l|)!(q+|l|)!}} \tag{8}$$

As discussed earlier, for instance, $HyGG_{2,l}$ beam can be expressed as the linear superposition of two $LG_{0,l}$ and $LG_{1,l}$ functions. According to Eqs. (7b), (7c) and (8) $HyGG_{2,l}$ beam is given by

$$|HyGG\rangle_{2,l} = \sqrt{\frac{l+1}{l+2}}|LG\rangle_{0,l} - \frac{1}{\sqrt{l+2}}|LG\rangle_{1,l} \tag{9}$$

It is noteworthy that the Gouy phase shift of $LG_{0,l}$ and $LG_{1,l}$ is