Optics & Laser Technology 79 (2016) 32–38

Contents lists available at ScienceDirect

Optics & Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Full length article

Study on propagation characteristics of temporal soliton in Scarff II PT-symmetric potential based on intensity moments

Yangbao Deng [a,*], Shuguang Deng [a,*], Chao Tan [b], Cuixiu Xiong [a], Guangfu Zhang [a], Ye Tian [a]

[a] College of Communication and Electronic Engineering, Hunan City University, Yiyang 413002, People's Republic of China
[b] School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411201, People's Republic of China

ARTICLE INFO

Article history:
Received 8 August 2015
Received in revised form
24 October 2015
Accepted 5 November 2015
Available online 28 November 2015

Keywords:
Temporal soliton
Propagation characteristics
Variable coefficient
Scarff II PT-symmetric potential

ABSTRACT

When a temporal soliton propagates in the inhomogeneous nonlinear medium with Scarff II parity-time (PT)-symmetric potential, we investigate the propagation characteristics of a temporal soliton based on intensity moments. Under the condition of Scarff II PT-symmetric potential, the propagation characteristics of a temporal soliton are affected by the dispersion coefficient, nonlinear coefficient and chirp. After a detailed analysis of the intensity evolution and the second-order intensity moment parameter, we find that the intensity and pulse width (PW) of a chirped-free temporal soliton are invariant during nonlinear propagation when the dispersion coefficients are the constant, exponential decreasing function and periodic modulated function, respectively. The intensity and PW of a chirped temporal soliton vary periodically when the dispersion coefficient is a periodic modulated function. So the chirp has no effect on propagation behavior of a temporal soliton. When the dispersion coefficients are the constant or exponential decreasing function, the intensity of a chirped temporal soliton is gradually increased, while the PW of a chirped temporal soliton is gradually decreased. Thus the temporal soliton is compressed and the chirp has a great effect on the propagation behavior of a temporal soliton. The results will be helpful to manipulation of nonlinear propagation of the laser pulses.

1. Introduction

In quantum mechanics, every physical observable is associated with a real spectrum, thus it must be Hermitian. However, Bender and co-works have pointed out that the non-Hermitian Hamiltonian with PT-symmetric can exhibit entirely real spectrum [1]. PT-symmetric indicates that the real part of a complex potential must be an even function and the imaginary part must be an odd function [1,2]. It was suggested that the refractive index distribution must be an even function and the gain/loss must be an odd function in optics [3]. In the past few years, Christodoulides et al. [4] creatively introduced the PT-symmetric into optics field and investigated the propagation characteristics of a beam in PT-symmetric potential. Regensburger et al. [5] firstly reported the synthetic PT-symmetric potential. Then lots of works focus on the PT-symmetric potential in theory and experiment. Many intriguing phenomena have been found, such as double refraction [6], power oscillation [6,7], unidirectional invisibility [8] and absorption enhanced transmission [9].

Optical soliton is a very fundamental and important nonlinear phenomenon in nature, which promise an important application for optical communication, optical storage, optical switch and all-optical information processing. Over the past decades, researchers have been investigated the variety of optical soliton in theory and experiment. For example, Mondal and Saha have been studied on the optical pulse propagating in different nonlinear medium and found some optical soliton [10–13]. Recently, Bhrawy and co-works have been done works in optical soliton for different system, and many optical soliton have been found [14,15]. Because the PT-symmetric medium has unique properties, which supports the PT optical soliton. Therefore, generation of a variety of PT soliton and analysis of the propagation stability of PT soliton in the area of PT-symmetric linear, mixed linear–nonlinear, and nonlinear potential have been investigated extensively [16–36].

Different-order intensity moments can describe the characteristics of a laser pulse, thus many researchers have been used the intensity moment method to analyze the propagation characteristics of a laser beam during linear propagation. The primary advantage of intensity moment method first proposed by Simon [37] is easy to obtain the output laser characteristics parameters according to the incident laser characteristics parameters. From the experimental view, only the intensity moments up to fourth-order make sense because the high-order intensity moments ($m+n>4$)

* Corresponding authors.
E-mail addresses: dyb5202008@aliyun.com (Y. Deng), shuguangdeng@163.com (S. Deng).

http://dx.doi.org/10.1016/j.optlastec.2015.11.003
0030-3992/

are subjected to large error and difficult to measure experimentally. The zero-order intensity moment shows the laser energy [38]. The gravity center of an optical field is described by the first-order intensity moment [39]. The second-order intensity moment is used to describe the beam width (BW), pulse width (PW), far-field divergence angle, M^2-factor, curvature radius and Rayleigh length [39–42]. Siegman has been proposed that the second-order intensity moment can characterize the laser beam quality [40]. Skewness is used to indicate the symmetry of a laser beam defined by the third-order intensity moment [39]. Kurtosis parameter represents the degree of sharpness of a laser beam defined by the fourth-order intensity moment [39,43–46].

In this paper, we investigate the propagation characteristics of a temporal soliton based on intensity moments when the temporal soliton propagates in an inhomogeneous nonlinear medium. Firstly, the temporal soliton solution is found by analytically solving the generalized nonlinear Schrödinger equation (GNLSE) with variable coefficient and Scarff II PT-symmetric potential. Then the intensity evolution of a temporal soliton is discussed. Finally, the propagation characteristics of a temporal soliton are analyzed in detail by calculating different-order intensity moments. This paper is organized as following. In Section 2, the model describing a laser pulse propagating in the Scarff II PT-symmetric potential is presented, and the temporal soliton solution is found. In Section 3, we study the intensity evolution of a temporal soliton during nonlinear propagation. In Section 4, the propagation parameters of a temporal soliton, such as gravity center, PW, skewness and kurtosis parameter are calculated based on intensity moments. Further, the propagation characteristics of a temporal soliton are analyzed in detail. Conclusions are presented in Section 5.

2. Model and temporal soliton solution

For the paraxial approximation, the $(1+1)D$ variable coefficient GNLSE with PT-symmetric potential has the form.

$$i\frac{\partial q}{\partial z} = -\frac{\beta(z)}{2}\frac{\partial^2 q}{\partial t^2} - \gamma(z)|q|^2 q - [v(z,t) + iw(z,t)]q, \tag{1}$$

where $q(z, x)$ is the complex envelope of the electrical field, t is the reduced time, z is the longitudinal propagation coordinate. The variable coefficients $\beta(z)$ and $\gamma(z)$ are the dispersion coefficient and nonlinear coefficient, respectively. $v(z, t)$ and $w(z, t)$ are the real and imaginary parts of the complex PT-symmetric potential, respectively.

In order to find the soliton solution of Eq. (1), the transformation is defined as

$$q(z,t) = \alpha(z)^{\frac{3}{2}} u(\xi,\tau)\exp[i\varphi(z,t)], \tag{2}$$

where

$$\tau = \alpha(z)t,\ \ \xi = \frac{\alpha(z)\Pi(z)}{2},\ \ \varphi(z,t) = -\frac{\alpha_0\alpha(z)}{2}t^2, \tag{3}$$

$$\alpha(z) = \frac{1}{\left[1 - \alpha_0\int_0^z \beta(\zeta)d\zeta\right]},\ \ \Pi(z) = \int_0^z \beta(\zeta)d\zeta. \tag{4}$$

Here $\alpha(z)$ is the temporal chirp function, which is related to $\beta(z)$. α_0 is the initial chirp value and $\Pi(z)$ is the accumulated dispersion. Then Eq. (1) becomes a simpler NLSE by substituting Eqs. (2)–(4) into Eq. (1).

$$i\frac{\partial u}{\partial \xi} = -\frac{\partial^2 u}{\partial \tau^2} - |u|^2 u - [V(\xi,\tau) + iW(\xi,\tau)]u, \tag{5}$$

The variable coefficients $\beta(z)$, $\gamma(z)$, $v(z,t)$ and $w(z, t)$ in Eq. (1) exist the following constraints.

$$\gamma(z) = \frac{\beta(z)}{2\alpha(z)}, \tag{6}$$

$$v(z,t) = \frac{\beta(z)\alpha^2(z)}{2}V(\tau), \tag{7}$$

$$w(z,t) = \frac{\beta(z)\alpha^2(z)}{2}W(\tau). \tag{8}$$

The $\gamma(z)$, $v(z, t)$ and $w(z, t)$ are depended on the $\beta(z)$ from Eqs. (6)–(8). The functions $v(z, t)$ and $w(z, t)$ must satisfy the restraint of even and odd functions. The Scarff II PT-symmetric potential is written as following [16].

$$V(\tau) = V_0\,\mathrm{sech}^2(\tau), \tag{9}$$

$$W(\tau) = W_0\,\mathrm{sech}(\tau)\tanh(\tau). \tag{10}$$

V_0 and W_0 are the modulation depth of the real and imaginary parts of the PT-symmetric potential. The corresponding linear problem associated with potential of Eqs. (9) and (10) exhibits an entirely real spectrum provided that $W_0 \le V_0 + 1/4$ [47]. From the Eqs. (9) and (10), it is easy found that the $V(\tau)$ are $W(\tau)$ satisfy the properties of PT-symmetric: $V(\tau)=V(-\tau)$ and $W(\tau)=-W(-\tau)$. The soliton solution to Eq. (1) can be obtained if the Eq. (5) is solved analytically. We seek the analytic solutions to Eq. (5) and assume the $u(\xi, \tau)$ in the form.

$$u(\xi,\tau) = A(\tau)\exp[i\lambda\xi + iB(\tau)], \tag{11}$$

where $A(\tau)$ are $B(\tau)$ are the real valued functions, λ is the corresponding real propagation constant. Substituting Eq. (11) into Eq. (5), $A(\tau)$ are $B(\tau)$ satisfy the following differential equations.

$$\frac{\partial^2 A(\tau)}{\partial \tau^2} - \left[\frac{\partial B(\tau)}{\partial \tau}\right]^2 A(\tau) + A^3(\tau) + V(\tau)A(\tau) = \lambda A(\tau), \tag{12}$$

$$\frac{\partial^2 B(\tau)}{\partial \tau^2}A(\tau) + 2\frac{\partial A(\tau)}{\partial \tau}\frac{\partial B(\tau)}{\partial \tau} + W(\tau)A(\tau) = 0. \tag{13}$$

A bound state nonlinear solution to Eqs. (12) and (13) must satisfy the localization condition $A \to 0$ as $\tau \to \infty$. Thus the $A(\tau)$ are $B(\tau)$ are solved.

$$A(\tau) = \sqrt{2 - V_0 + \frac{W_0^2}{9}}\,\mathrm{sech}(\tau), \tag{14}$$

$$B(\tau) = \frac{W_0}{3}\arctan[\sinh(\tau)]. \tag{15}$$

The propagation constant λ is solved as 3. The complex Scarff II PT-symmetric potential with variable coefficient are presented by substituting the Eqs. (9)–(10) into Eqs. (7)–(8).

$$v(z,t) = \frac{\beta(z)\alpha^2(z)}{2}V_0\,\mathrm{sech}^2[\alpha(z)t], \tag{16}$$

$$w(z,t) = \frac{\beta(z)\alpha^2(z)}{2}W_0\,\mathrm{sech}[\alpha(z)t]\tanh[\alpha(z)t]. \tag{17}$$

After substituting the Eqs. (11), (14) and (15) back into Eq. (2), the temporal soliton solution of the variable coefficient GNLSE with Scarff II PT-symmetric potential is found.