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Shaping of spherical light intensity based on the interference of tightly focused beams with different polarizations

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ABSTRACT

We consider the shaping of spherical intensity distributions based on the interference of counter-propagating tightly focused vortex beams with different polarizations. The formation of 3D distributions is performed using a simple method of optimization of the width and position of the single annular aperture. The optimum parameters for the narrow aperture are calculated analytically. In addition the wide aperture parameters are numerically adjusted. It is shown that depending on the polarization, the additional vortex phase and/or phase shift in the beams allow to form either solid light balls or light spheres of subwavelength radius. They consist of the various electric field components.

1. Introduction

The three-dimensional shaping of tightly focused laser radiation is an actual problem in optical trapping, micro-manipulation, microscopy and data recording [1–3]. Tight focusing with high-aperture lenses is used to reduce the transverse dimension of the focal spot. However, the longitudinal size of the focal spot $\Delta z = \lambda n/NA^2$ (λ is wave length, n is refractive index of the medium, and NA is numerical aperture of focusing system) is two times higher than the transverse one $\Delta r = \lambda/(2NA)$ even in the limiting case of $NA = 1$ for $n = 1$.

Three-dimensional analysis of the generated field is greatly simplified when the contribution of one of the electric field components is much higher than the others. There are various ways to achieve this practically scalar mode. In particular, for the radial polarization case it is the narrow annular aperture [4] that transmits only the peripheral rays or an additional axicon [5] that maximizes the contribution of the longitudinal component. However, in both cases, the transverse dimension of the focal spot is further reduced and the longitudinal size is increased.

Different methods have been proposed to solve this problem. Most of them are based on the interference effect [6–9]. Furthermore, in this case it is possible to annihilate the contribution of certain components in the focal (or other certain) plane using the interference of counter-propagating beams.

Currently there are many optical schemes that implement a particular type of interference of the counter-propagating beams. Among them there are several basic types that use the confocal lens system or mirror reflection [8–10].

In each case the selective addition or subtraction of the respective components of the electromagnetic field of the counter-propagating beams in the defined plane can be achieved by phase modulation [11,12], choice of polarization [13,14], or specific refractive and reflective elements [9,15,16].

In this paper we consider the formation of spherical intensity distributions based on the interference of tightly focused vortex beams with different polarizations. It is shown that depending on the polarization, order of the vortex singularity and additional phase shift in the beams either solid balls of light or spheres of subwavelength radius can be formed. Change of parameters of the beams allows to form spherical distributions with the various electric field components.

2. Component analysis of tightly focused vortex electric field

In the case of tight focusing, the electric field in the focal plane is described by the following expression:

$$\mathbf{E}(\rho,\varphi,z) = -\frac{if}{\lambda}\int_0^{\alpha}\int_0^{2\pi} B(\theta,\phi)T(\theta)\mathbf{P}(\theta,\phi)\exp[ik(r\sin\theta\cos(\phi-\varphi) + z\cos\theta)]\sin\theta\, d\theta\, d\phi, \quad (1)$$

where (ρ,φ,z) are the cylindrical coordinates in the focal region, (θ,ϕ) are the spherical angular coordinates of the focusing system's output pupil, α is the maximal value of the azimuthal angle related to the system's NA, $B(\theta,\phi)$ is the transmission function, $T(\theta)$ is the pupil's apodization function (is equal $\sqrt{\cos\theta}$ for aplanatic systems), $\mathbf{P}(\theta,\phi)$ is the polarization vector,

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$k=2\pi/\lambda$ is the wave number, λ is the wavelength, and f is the focal length.

The vector $\mathbf{P}(\theta,\phi)$ for the Cartesian components is

$$\mathbf{P}_d(\theta,\phi)=\begin{bmatrix}1+\cos^2\phi(\cos\theta-1) & \sin\phi\cos\phi(\cos\theta-1) & \cos\phi\sin\theta\\ \sin\phi\cos\phi(\cos\theta-1) & 1+\sin^2\phi(\cos\theta-1) & \sin\phi\sin\theta\\ -\sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta\end{bmatrix}\begin{pmatrix}c_x(\phi)\\ c_y(\phi)\\ c_z(\phi)\end{pmatrix} \tag{2}$$

Using the connection between the Cartesian and cylindrical projections,

$$\begin{pmatrix}\mathbf{e}_r\\ \mathbf{e}_\phi\\ \mathbf{e}_z\end{pmatrix}=\begin{bmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{pmatrix}\mathbf{e}_x\\ \mathbf{e}_y\\ \mathbf{e}_z\end{pmatrix}, \tag{3}$$

the vector $\mathbf{P}(\theta,\phi)$ can be written for the cylindrical components as follows:

$$\mathbf{P}_c(\theta,\phi)=\begin{bmatrix}\cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta\\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}\begin{pmatrix}c_r(\phi)\\ c_\phi(\phi)\\ c_z(\phi)\end{pmatrix}. \tag{4}$$

The representation (2) is convenient for linear polarization, and expression (4) is better for the radial, azimuthal and circular polarizations. Due to the task of forming the spherical distribution in the focal plane, the linear polarization will not be considered, as it is not symmetric in the transverse plane in tightly focusing.

For the vortex beams $B(\theta,\phi)=G(\theta)\exp(im\phi)$ Eq. (1) is simplified [17] as follows:

$$\mathbf{E}_m(\rho,\varphi,z)=-ikf\int_0^\alpha G(\theta)T(\theta)\mathbf{Q}_m(\rho,\varphi,\theta)\sin\theta\exp(ikz\cos\theta)d\theta, \tag{5}$$

where the components of $\mathbf{Q}_m(\rho,\varphi,\theta)$ are the sums of the Bessel functions of different order and depend on the polarization vector $\mathbf{c}(\phi)$ of the incident beam.

In particular, the radial polarization vector $\mathbf{Q}_m(\rho,\varphi,\theta)$ in Eq. (5) is as follows:

$$\mathbf{Q}_m^{rad,c}(\rho,\varphi,\theta)=\frac{i^m e^{im\varphi}}{2}\begin{bmatrix}i[J_{m+1}(k\rho\sin\theta)-J_{m-1}(k\rho\sin\theta)]\cos\theta\\ [J_{m+1}(k\rho\sin\theta)+J_{m-1}(k\rho\sin\theta)]\cos\theta\\ -2J_m(k\rho\sin\theta)\sin\theta\end{bmatrix}, \tag{6}$$

while for the azimuthal polarization it is

$$\mathbf{Q}_m^{az,c}(\rho,\varphi,\theta)=-\frac{i^m e^{im\varphi}}{2}\begin{bmatrix}J_{m+1}(k\rho\sin\theta)+J_{m-1}(k\rho\sin\theta)\\ -i[J_{m+1}(k\rho\sin\theta)-J_{m-1}(k\rho\sin\theta)]\\ 0\end{bmatrix}, \tag{7}$$

and for the circular «$\pm$» polarization it is

$$\mathbf{Q}_m^{circ\pm,c}(\rho,\varphi,\theta)=\frac{i^m e^{i(m\pm1)\varphi}}{\sqrt{2}}\begin{bmatrix}J_m(k\rho\sin\theta)-\frac{1}{2}[J_m(k\rho\sin\theta)-J_{m\pm2}(k\rho\sin\theta)](1-\cos\theta)\\ \pm i\{J_m(k\rho\sin\theta)-\frac{1}{2}[J_m(k\rho\sin\theta)+J_{m\pm2}(k\rho\sin\theta)](1-\cos\theta)\}\\ \mp iJ_{m\pm1}(k\rho\sin\theta)\sin\theta\end{bmatrix}. \tag{8}$$

Eq. (5) simplifies the analysis in the three-dimensional structure of the focal field, however it should be kept in mind that the formation of a spherical distribution in the focal area (ball or sphere) requires non-zero energy on the optical axis (intensity distribution would otherwise be open along the optical axis). Thus, for the considered types of polarizations the order of vortex phase function should be $|m|\le 2$.

3. Mirror interference of counter-propagating beams

Currently for various optical schemes for performance a particular type of counter-propagating beams interference is developed. Among them there are several basic types that use the confocal lens system or mirror reflection [8–10].

A scheme which allows to add the transverse components of two beams and to subtracting the longitudinal components is described in [8]. If one of the beams has a π-phase shift, then on the contrary, the longitudinal components will be added, and the transverse ones will be subtracted [9,11,12].

Interference due to mirror reflection (see Fig. 1) can be generally represented using the following expression:

$$\mathbf{E}_{mr}^d(x,y,z)=\mathbf{E}_1(x,y,z)+\exp(ib)\mathbf{E}_2(x,y,z)$$
$$=\begin{pmatrix}E_{1x}(x,y,z)+\exp(ib)E_{1x}(x,y,-z)\\ E_{1y}(x,y,z)+\exp(ib)E_{1y}(x,y,-z)\\ E_{1z}(x,y,z)-\exp(ib)E_{1z}(x,y,-z)\end{pmatrix}. \tag{9}$$

If $b=\pi$, signs in (9) are reversed.

Taking into account Eq. (3), expression (9) for cylindrical coordinates will look similar. Using Eq. (1) interference (9) in cylindrical coordinates is described by the following expression (if $b=0$):

$$\mathbf{E}_{mr}^c(r,\varphi,z)=-\frac{2if}{\lambda}\int_0^\alpha\int_0^{2\pi}B(\theta,\phi)T(\theta)\begin{pmatrix}P_{1r}(\theta,\phi)\cos(kz\cos\theta)\\ P_{1\varphi}(\theta,\phi)\cos(kz\cos\theta)\\ P_{1z}(\theta,\phi)\sin(kz\cos\theta)\end{pmatrix}\exp[ikr\sin\theta\cos(\phi-\varphi)]\sin\theta\,d\theta\,d\phi, \tag{10}$$

where the components of vector $\mathbf{P}_1(\theta,\phi)$ are determined according to Eq. (4).

If $b=\pi$, the destructive and constructive interference in expression (10) for the various components of the electric vector will be interchanged.

Note that Eq. (10) can be reduced to form (5), which makes the calculations and theoretical analysis easier. In the following sections the analysis of interference (9) for the selected polarization types for vortex beams is performed.

4. The interference of axially symmetric beams with different polarizations

This case corresponds to the $m=0$ in expression (5) and it is the most attractive from the practical point of view (because creating a vortex phase requires additional optical elements).

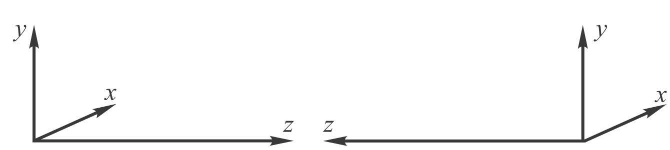

Fig. 1. Coordinate systems for counter-propagating beam interference due to mirror reflection.