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Propagation of a stochastic electromagnetic vortex beam in the oceanic turbulence

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ABSTRACT

By using the extended Huygens–Fresnel principle, we investigate the stochastic electromagnetic vortex beam propagating through oceanic turbulence. General formulas for the elements of the 2×2 cross-spectral density matrix of a stochastic electromagnetic vortex beam propagating through the oceanic turbulence are obtained. We study the changes in the spectral density, the spectral degree of coherence and the spectral degree of polarization of such a vortex beam with the help of the general formulas. It is shown by numerical calculations that, the beam profile will approach a Gaussian distribution in far field under the influence of oceanic turbulence. It is also interesting to find that the spectral degree of polarization of a stochastic electromagnetic vortex beam composed by isotropic sources on propagation in far zone will return to its value in the source plane.

1. Introduction

The unified theory of coherence and polarization of a stochastic electromagnetic beam on propagation was presented by Wolf in 2003 [1,2]. Since then a lot of work has been done to discuss the properties of a stochastic electromagnetic laser beam on propagation both in free space [3,4], through deterministic media (such as optical fibers [5], chiral media [6], optical systems [7] and compound photonic crystals [8], etc.), and through random media (just like atmospheric turbulence [9,10], tissues [11,12], and oceanic turbulence [13–15]).

Since Nye and Berry firstly introduced the concept of vortex beam on an optical wave field in 1974 [16], there has been substantial interests concerning optical vortex because of its interesting properties [17–19] and wide applications [20–22]. In recent years, propagation of vortex beams in the turbulent atmosphere has been investigated [23]. Statistical properties of stochastic electromagnetic vortex beams propagating in free space have also been discussed in detail [24]. However, to the best of our knowledge, the propagation properties of stochastic electromagnetic vortex beam propagating in the oceanic turbulence have not been studied and reported.

In this paper, we derive a general formula for the elements of cross-spectral density matrix of a stochastic electromagnetic vortex beam while propagating in the oceanic turbulence with the help of the unified theory of coherence and polarization of stochastic electromagnetic beams. The spectral density, the spectral degree of coherence and the spectral degree of polarization of such a beam are investigated in detail.

2. Theoretical analyses

The electric field of a stochastic electromagnetic vortex beam at the source plane (i.e. $z=0$) can be expressed as [24]

$$\mathbf{E}(\mathbf{r},\omega,z=0)=A(\mathbf{r},\omega,z=0)\exp(im\phi), \tag{1}$$

where $A(\mathbf{r},\omega,z=0)$ is the statistical ensemble of fluctuating electric field in the source plane, m is the topological charge of the vortex, and ω is the angular frequency, and the $\exp(im\phi)$ being the phase factor. For computational convenience and without loss of generality in the following discussion, we assume that the source field amplitude of the optical vortex is a Laguerre–Gaussian mode [24,25]:

$$A(\mathbf{r},z=0)=E_0(r/\sigma)^m\exp(-r^2/\sigma^2)\exp(i\beta), \tag{2}$$

where E_0 and σ are the characteristic amplitude and beam size in the source plane, respectively, and β is an arbitrary phase.

Consider a stochastic electromagnetic vortex beam propagating close to the z axis from the source plane (i.e.$z=0$) to the half-space $z\geq0$ in an oceanic turbulence. The second-order coherence and polarization properties of a stochastic, statistically stationary electromagnetic beam at a pair of points $\mathbf{r}_1,\mathbf{r}_2$ may be characterized by the 2×2 electric cross-spectral density matrix [1]

$$\overleftrightarrow{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv[W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega)]=[\langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle],\quad (i=x,y;\ j=x,y), \tag{3}$$

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where the asterisk denotes the complex conjugate and the angular brackets represent the average over the ensemble of realizations of the fluctuating electric field. x and y are two mutually orthogonal directions perpendicular to the beam axis. $E_i(\mathbf{r}_1,\omega)$ and $E_j(\mathbf{r}_2,\omega)$ are Cartesian components of frequency component ω of the complex electric vector at a point specified by the transverse position vector $\mathbf{r}$.

When a stochastic electromagnetic vortex beam propagates in the oceanic turbulence, the cross-spectral density of such a beam in the output plane $z>0$ can be obtained from the knowledge of the cross-spectral density in the source plane $z=0$ with the help of the extended Huygens–Fresnel integral [26], viz.,

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) &= (\frac{k}{2\pi z})^2 \iint d\mathbf{r}_1 \iint d\mathbf{r}_2 W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\\ z=0)&\exp[-ik\frac{(\boldsymbol{\rho}_1-\mathbf{r}_1)^2-(\boldsymbol{\rho}_2-\mathbf{r}_2)^2}{2z}]\\ &\times\exp[-\frac{\pi^2k^2z}{3}(\mathbf{r}_1-\mathbf{r}_2)^2]\int\kappa^3\Phi_n(\kappa)d\kappa\\ &=(\frac{k}{2\pi z})^2\iint d\mathbf{r}_1\iint d\mathbf{r}_2 W_{ij}(\mathbf{r}_1,\mathbf{r}_2,z=0)\exp[-ik\frac{(\boldsymbol{\rho}_1-\mathbf{r}_1)^2-(\boldsymbol{\rho}_2-\mathbf{r}_2)^2}{2z}]\\ &\times\exp[-\frac{1}{{\rho_0}^2}(\mathbf{r}_1-\mathbf{r}_2)^2]. \end{aligned} \tag{4}$$

Here, $k=2\pi/\lambda$ is the wave number with λ being the wavelength, z is the propagation distance, $\boldsymbol{\rho}_1,\boldsymbol{\rho}_2$ are the position vectors in the output plane, and ρ_0 is the coherence length of a spherical wave propagating in the turbulent medium which can be expressed as

$$\rho_0=\left(\frac{3}{\pi^2k^2z\int_0^\infty\kappa^3\Phi_n(\kappa)d\kappa}\right)^{1/2} \tag{5}$$

The model we used for the spatial power spectrum of the refractive index fluctuations of the oceanic water was obtained in [14,27], as a linearized polynomial of two variables: the temperature fluctuations and the salinity fluctuations. The model is valid under the assumption that the turbulence is isotropic and homogeneous and, hence, we require only specification of the one-dimensional spectrum, which has the form

$$\Phi_n(\kappa)=0.388\times10^{-8}\varepsilon^{-1/3}\kappa^{-11/3}[1+2.35(\kappa\eta)^{2/3}]f(\kappa,w,\chi_T), \tag{6}$$

here ε is the rate of dissipation of turbulent kinetic energy per unit mass of fluid which may vary in range from 10^{-4} to $10^{-10}\mathrm{m}^2/\mathrm{s}^3$, $\eta=10^{-3}$m being the Kolmogorov microscale (inner scale), with

$$f(\kappa,w,\chi_T)=\frac{\chi_T}{w^2}(w^2e^{-A_T\delta}+e^{-A_S}\delta-2we^{-A_{TS}\delta}), \tag{7}$$

and χ_T being the rate of dissipation of mean-square temperature, $A_T=1.863\times10^{-2}$, $A_s=1.9\times10^{-4}$, $A_{TS}=9.41\times10^{-3}$, and $\delta=\ 8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2$, w being the relative strength of temperature and salinity fluctuations, where in the ocean water can vary from -5 to 0, 0 value corresponding to the case when temperature-driven turbulence dominates, -5 value corresponding to the situation when salinity-driven turbulence prevails.

On substituting from Eqs. (1) and (2) into Eq. (3), assuming that the statistical distribution of β corresponding to a Gaussian–Schell correlator, and the off-diagonal elements of the electric cross-spectral density matrix of the beam in the source plane have zero value (i.e. $W_{xy}(\mathbf{r}_1,\mathbf{r}_2,z=0)=W_{yx}(\mathbf{r}_1,\mathbf{r}_2,z=0)=0$). Considering the above assumption, the elements of cross-spectral density matrix of such a beam in the source plane $z=0$ can be written as [24]

$$\begin{aligned} W_{ii}(\mathbf{r}_1,\mathbf{r}_2,z=0)&=I_{i0}(r_1/\sigma_i)^{m_i}(r_2/\sigma_i)^{m_i}\exp\left[-\frac{r_1^2+r_2^2}{\sigma_i^2}\right]\exp[-im_i(\phi_1-\phi_2)]\\ &\times\exp\left[-\frac{r_1^2+r_2^2-2r_1r_2\cos(\phi_1-\phi_2)}{\delta_{ii}^2}\right], \end{aligned} \tag{8}$$

where $I_{i0}=E_{i0}^2$.

On substituting from Eq. (8) into Eq. (4), we obtain

$$\begin{aligned} W_{ii}(\rho_1,\rho_2,\theta_1,\theta_2,z)&=I_{i0}^2\left(\frac{k}{2\pi z}\right)^2\exp\left[-\frac{ik}{2z}(\rho_1^2-\rho_2^2)\right]\\ \iiint\!\!\!\!\!\!\int&\exp\left[-\frac{ik}{2z}(r_1^2-r_2^2)\right]\times\left(\frac{r_1}{\sigma_i}\right)^2\left(\frac{r_2}{\sigma_i}\right)^2\exp\left[-\left(\frac{1}{\sigma_i^2}+\frac{1}{\delta_{ii}^2}+\frac{1}{\rho_0^2}\right)(r_1^2+r_2^2)\right]\\ &\times\exp\left[\frac{ikr_1\rho_1}{z}\cos(\theta_1-\phi_1)\right]\exp\left[-\frac{ikr_2\rho_2}{z}\cos(\theta_2-\phi_2)\right]\\ &\times\exp\left[2r_1r_2\cos(\phi_1-\phi_2)\left(\frac{1}{\delta_{ii}^2}+\frac{1}{\rho_0^2}\right)\right]\\ &\times\exp[-im_i(\phi_1-\phi_2)]r_1r_2dr_1dr_2d\phi_1d\phi_2. \end{aligned} \tag{9}$$

Using the following equations [28]

$$\exp\left[\frac{ik\rho r}{z}\cos(\theta-\phi)\right]=\sum_{l=-\infty}^{\infty}i^lJ_l\left(\frac{k\rho r}{z}\right)\exp[il(\theta-\phi)], \tag{10}$$

$$\int_0^{2\pi}\exp\left[-im\phi_1+\frac{2r_1r_2}{\delta^2}\cos(\phi_1-\phi_2)\right]d\phi_1=2\pi\exp(-im\phi_2)I_m\left(\frac{2r_1r_2}{\delta^2}\right), \tag{11}$$

$$\int_0^{2\pi}\exp(in\phi)d\phi=\begin{cases}2\pi & (n=0),\\ 0 & (n\neq0),\end{cases} \tag{12}$$

After tedious calculation, Eq. (9) can be simplified as

$$\begin{aligned} W_{ii}(\rho_1,\rho_2,\theta_1,\theta_2,z)&=I_{i0}^2\left(\frac{k}{z}\right)^2\exp\left[-\frac{ik}{2z}(\rho_1^2-\rho_2^2)\right]\sum_{l=-\infty}^{\infty}\\ \iint&\exp\left[-\frac{ik}{2z}(r_1^2-r_2^2)\right]\times\left(\frac{r_1}{\sigma_i}\right)^2\left(\frac{r_2}{\sigma_i}\right)^2\exp\left[-\left(\frac{1}{\sigma_i^2}+\frac{1}{\delta_{ii}^2}+\frac{1}{\rho_0^2}\right)(r_1^2+r_2^2)\right]\\ &\times J_l\left(\frac{k\rho_1r_1}{z}\right)J_l\left(\frac{k\rho_2r_2}{z}\right)I_{l+m_i}\left[2r_1r_2\left(\frac{1}{\delta_{ii}^2}+\frac{1}{\rho_0^2}\right)\right]\\ &\times\exp[-il(\theta_1-\theta_2)]r_1r_2dr_1dr_2. \end{aligned} \tag{13}$$

Now let $\rho_1=\rho_2=\rho$, $\theta_1=\theta_2=\theta$, the spectral density and the degree of polarization of the beam at the point $(\boldsymbol{\rho},z)$ in the propagation field are given by

$$S(\boldsymbol{\rho},z,\omega)=\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},z)=W_{xx}(\rho,\rho,\theta,\theta,z)+W_{yy}(\rho,\rho,\theta,\theta,z), \tag{14}$$

$$P(\boldsymbol{\rho},z;\omega)=\sqrt{1-\frac{4\mathrm{Det}[\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},z)]}{\mathrm{Tr}[\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},z)]^2}}, \tag{15}$$

where Det and Tr stand for the determinant and trace of the matrix, respectively.

By letting $\rho_1=-\rho_2=\rho$, the spectral degree of coherence of the electric field at a pair of points $(\boldsymbol{\rho}_1,z)$ and $(\boldsymbol{\rho}_2,z)$ is given by the formula

$$\mu(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z,\omega)=\frac{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho},-\boldsymbol{\rho},z,\omega)}{\sqrt{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},z,\omega)}\cdot\sqrt{\mathrm{Tr}\overleftrightarrow{W}(-\boldsymbol{\rho},-\boldsymbol{\rho},z,\omega)}} \tag{16}$$

Based on Eqs. (13)–(16), we can perform some numerical calculations of a stochastic electromagnetic vortex beam propagating through turbulent ocean which are shown in the next section.