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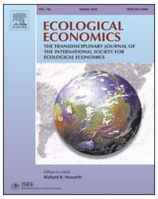

A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources☆

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we survey the literature applying viability theory to the sustainable management of renewable resources. After a refresher on the main concepts of viability theory, we provide a general map of the contributions and next discuss them by area of application, including ecosystems and population biology, climate change, forestry and others. We conclude by pointing out issues that deserve more attention and should be part of a research agenda.

1. Introduction

It is not new that societies care about their environment and resources and take actions to protect them.[1] What is however of recent vintage is the awareness that (i) immoderate human activity, e.g., burning fossil fuels, over fishing or excessive deforestation, have has direct undesirable consequences, such as loss of biodiversity and deterioration in environmental quality, and (ii) some concerted actions are urgently needed to preserve these resources. A pivotal date in first gaining this awareness was probably the publication of *Limits of Growth* in 1972 (Meadows et al., 1972), a study that triggered fervent debate and stroked the popular imagination, since some of the simulated growth scenarios predicted the collapse of the global system. Later in the same decade, it was argued that economic development could be sustained indefinitely, but only if it were was to take into account its ultimate interaction with the natural environment. This marked the advent of the concept of *ecological management*, which paved the way for the notion of *sustainable development*, which was coined by the International Union for the Conservation of Nature and Natural Resources (IUCN) in 1980; see (Allen et al., 1980). Although at that time a precise definition of sustainable development was lacking, the idea itself very quickly gained in popularity among scientists, decision makers and activists.[2] A second notable date is the publication in 1987 of the Brundtland Report, which provided a unifying definition of sustainable development:

> "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs."
>
> (Brundtland et al., 1987)

This definition has since been adopted by all stakeholders, although refinements have occasionally been considered, implicitly or explicitly, in some studies (See see for example Pezzey (1992), Neumayer (2003), Heal (1998) and Klauer (1999) for an overview of some characterizations and operationalizations of sustainability that have been proposed). For example, Fleurbaey (2015) proposed to define sustainability in terms of leaving the possibility for future generations to sustain certain defined targets. Martinet et al. (2007) defined sustainability as a combination of biological, economic and social constraints which need to be met. Baumgärtner and Quaas (2009) conceptualized strong sustainability under uncertainty as ecological-economic viability. Durand et al. (2012) and Doyen and Martinet (2012) considered the notion of intergenerational equity in defining sustainability.

This paper provides a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources including ecosystems and populations such as fisheries and non-marine species, the environment (with a focus on climate change and GHG concentration), and other resources (e.g., forests and soil). In a nutshell, "Viability theory is an area of mathematics that studies the evolution of dynamical systems under constraints on the

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[1] The following website offers an environmental history timeline with a list of events and actions related to environmental protection: http://environmentalhistory.org.

[2] For a list of some definitions of sustainable development used in between 1980 and 1988, see the Appendix in Pezzey (1992).

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system's state and control (Aubin, 1991b; Aubin et al., 2011). It was developed to formalize problems arising in the study of various natural and social phenomena, and has close ties with the theories of optimal control and set-valued analysis."[3] As in optimal control, the basic ingredients of viability theory (VT) are control and state variables, and a dynamical system whose evolution is governed by differential (or difference) equations, which are functions of the state and control variables and some parameters. The system evolution can be deterministic or not, and is subject to some (viability) constraints. A notable difference with optimal control is the absence of an objective functional to be optimized. As we will see, the main objects of viability theory are sets, hence the link made above to set-valued analysis. The theory was initiated by Jean-Pierre Aubin in the late 1970s and the fundamental results established in the 1980s (see Haddad, 1981).

In Aubin (1991a), viability theory is described as a mathematical theory based on three main features, namely: (i) non-determinism of evolutions; (ii) viability constraints; and (iii) inertia principle. The two first features concern the state trajectory of the studied system and reflect the fact that a system can evolve in many different and possibly unpredictable ways depending on its initial state, its past evolution, the environment in which it evolves or anything else (non determinism non-determinism), and also the fact that, for many reasons, the evolution of a system is restrained by some constraints that must be satisfied at each instant of time[4]. These are the two founding pillars of viability theory models.[5] The last feature (inertia principle) concerns the control variables and stipulates that these controls are changed only when required for maintaining viability. To find a viable solution (or a set of viable solutions), VT follows a backward (or inverse) method, that is, starting from a set of given viability constraints, one looks for the set of initial states from which the system can be indefinitely viable.

Viability theory was successfully applied in many fields, including economics (Aubin, 1997), finance (Aubin et al., 2005b), demography and genetics (Bonneuil and Saint-Pierre, 2000; Bonneuil and Saint-Pierre, 2008), aerospace (Tomlin et al., 2003) and in renewable resources management, which is our topic. Other approaches than VT are of course available to determine sustainable exploitation of a renewable resource, in particular the so-called *policy optimization* and *policy evaluation* (Weyant et al., 1996). In the former, as the name suggests, one defines an objective function that typically measures the relevant costs and benefits of possible decisions, and the optimization is carried out subject to a series of constraints. In policy evaluation, some feasible scenarios are assessed and eventually the best one is selected. While these approaches have obvious merits, they often involve trade-offs between the different environmental, economic and social facets of sustainability, which may not be desirable. As mentioned above, there is no (intertemporal) objective to be optimized in a VT model, and sustainability is addressed through the viability constraints. Therefore, a VT model avoids the contentious issue of weighting different sustainability facets, or making trade-offs between short- and long-term considerations. Writing down an intertemporal objective requires an assessment of future options. In a VT model, such knowledge of the future is not mandatory because the choice of controls at any given initial time is not final, and can be adapted to eventual changes in the system's environment (Aubin, 1990). It is generally difficult to compute viable controls in a closed form. However numerical methods can be used to approximate the viability kernels and viable controls. This is somehow similar to what is done in the *policy guidance approach* (PGA), which was recently proposed and has been referred to by different names in different areas, e.g., *tolerable window approach* in climate change and GHG management (Scientific Advisory Council on Global Change, 1995; Bruckner et al., 1999, 2003; Toth et al., 2002), *population viability analysis* in conservation biology (Beissinger and Westphal, 1998; Ferrière and Baron, 1996; Ellner et al., 2003; Boyce, 1992; Beissinger and McCullough, 2002; Shaffer, 1990; Beissinger, 2002) or *safe minimum standards* in fisheries (Berrens, 2001; Berrens et al., 1998; Bishop, 1980). Indeed, the basic idea behind the PGA is to maintain the system as long as possible within some predefined bounds (De Lara and Doyen, 2008). Finally, we note that determining feedback control maps when solving a VT model is similar to what is done when solving a dynamic optimization problem using dynamic programming.

The rest of the paper is organized as follows: In Section 2, we provide a short refresher on viability theory. In Section 3, we review the applications of viability theory to the management of renewable resources, which is the main block of interest. In Section 4, we briefly conclude. A table summarizing all reviewed papers is given in the Appendix.

2. A Refresher on Viability Theory

In this section, we recall some concepts of viability theory that are useful for appreciating its applications in renewable resources. For a rigorous introduction to viability theory, the interested reader may consult the books by Aubin (1991a,b), Aubin et al. (2011) and De Lara and Doyen (2008).

We shall distinguish in the sequel between deterministic viability and stochastic viability. Although in both settings the main questions are the same, e.g., how to remain viable, to reach a target or to restore viability if lost during the process, the concepts and techniques used to answer these questions will be different, at least to a certain extent.

Denote by $x(t)$ the state of a system of interest at time $t \in [0, +\infty)$, and let $X \subset \mathbb{R}^n$ be the state space. The evolution of the state is described by

$$\mathscr{F}\begin{cases} x'(t) = f(x(t), u(t)) \\ u(t) \in U(x(t)) \end{cases}, \tag{1}$$

where $u(\cdot)$ is the control variable and $U(x(t))$ is the set of admissible controls at time t, which depends on the state of the system at that time. We shall refer to $\mathscr{F}$ as the controlled-evolution system.

At each time t and starting from any state x, the system can follow different trajectories depending on the applied control u and other parameters. We denote by $\mathscr{S}$ the set of all solutions of the system (1) and $\mathscr{S}(x) \subset \mathscr{S}$ the set of all admissible trajectories starting from x and governed by Eq. (1), that is,

$$\mathscr{S}(x) = \{x(\cdot) | x(0) = x \quad \text{and Eq. (1) satisfied}\}.$$

where $x(\cdot)$ are absolutely continuous functions.

Let $K \subset X$ be the set of (viability) constraints. In its simplest expression, this set would involve lower and upper bounds on the state variables, i.e.,

$$K = \left\{ x \in X | \vdots \begin{matrix} \underline{x}_1 \le x_1 \le \overline{x}_1 \\ \\ \underline{x}_n \le x_n \le \overline{x}_n \end{matrix} \right\},$$

but of course, in general, the constraints can be more complex, i.e., of the form:

$$K = \left\{ x \in X | \vdots \begin{matrix} g_1(x) \ge 0 \\ \\ g_m(x) \ge 0 \end{matrix} \right\}.$$

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[3] https://en.wikipedia.org/wiki/Viability_theory.

[4] When the model is stochastic, satisfying the constraints at each instant of time has to be interpreted in a stochastic or robust-control sense.

[5] Besides, Aubin et al. (2011) present this theory as a mathematical translation of Jacques Monod's *Chance and Necessity* (Monod, 1971) in which there appears a quotation from Democritus stating that "the whole universe is but the fruit of two qualities, chance and necessity." Chance refers to the non-determinism of trajectories, and necessity expresses the need to meet certain conditions or criteria, which results in viability constraints.