Optics & Laser Technology 48 (2013) 503–508

Contents lists available at SciVerse ScienceDirect

Optics & Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Application of the Fourier analysis methods to the three beam interferometry

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ARTICLE INFO

Article history:
Received 15 June 2012
Received in revised form
19 November 2012
Accepted 25 November 2012
Available online 23 December 2012

Keywords:
Optical vortex
Interferometry
Fourier analysis

ABSTRACT

In this paper, method for enhancing the measurement capability of an optical vortex interferometer is proposed. We adapted the techniques used in the carrier frequency interferometry to describe a way of measuring the change of the object's beam propagation direction and of reconstructing the geometry of its wavefront. To do this the interference field of the three beam interferogram and its Fourier spectrum has been calculated. The experimental results obtained with the presented technique are in agreement with the results obtained by using the analysis applied to the optical vortex interferometer. The proposed method does not require any alterations in the optical vortex interferometer body. The calculations are performed on the single three-beam interferogram which is fundamental for the optical vortex interferometry. In this way we take advantage of both, the carrier frequency as well as the optical vortex interferometry.

1. Introduction

Interference fringes are a well known pattern characteristic of an two-beam interferometry. A lesser known fact is that an interference of the non-coplanar three-plane waves generates a regular lattice of optical vortices [1–2]. Disturbance of one, two or all three waves causes the change in the vortex lattice geometry [3–5]. In many cases a mathematical relation holds between the vortex lattice change and physical parameters of the sample introduced into the interfering waves. An optical vortex interferometer (OVI) utilizes such relations for metrological purposes. An OVI can work with more than three waves. However, as shown in [6], the vortex lattice produced by the interference of four or more plane waves loses some of its special properties helpful in vortex lattice analysis. On the other hand, replacing plane waves with spherical ones is possible without losing the vortex lattices special properties [7–9]. Compact polarization versions of OVI were reported in [10–12].

The advantages [6] of the three waves OVI mentioned above resulted from the fact that the ends of three non-collinear wavevectors (representing the three interfering waves) define a plane. The optical vortices propagate along straight lines perpendicular to this plane. This is a very typical behavior. Adding more waves makes the vortex trajectories much more complicated, unless very specific conditions are met [13]. The three interfering waves are denoted as *A*, *B* and *C*. The C wave is an object wave. The special properties of the three beam vortex lattice are as follows: (a) the vortex lattice moves like a rigid body when changing the phase of any of *A*, *B* or *C* waves, (b) the value of these phase shifts do not have to be equal, (c) the distribution of negative (positive) vortices is independent on the amplitudes of the interfering waves, contrary to the distance between negative and positive vortex sublattices, and (d) all parameters describing vortex lattice geometry can be described with simple analytical formulas. The three-beam OVI advantages open up the new possibilities in interferometry. Two examples are given below: (a) with a single measurement the angle of the object wave rotation and its decomposition against two perpendicular axes (x and y) can be determined and (b) the wavefront geometry can be reconstructed without ambiguity (phase unwrapping problem), provided that the vortices signs are known [5,14,15]. There are many ways of finding this sign in any OVI configuration [16,17].

There are two weak points of OVI: limited precision of the vortex points localization and the fact that only 300–400 points occupied by vortex points can be used. Typically the image taken by a CCD camera consists of about 1 million pixels and most of them are not used. The examples of effective localization procedures are given in [14]. The second problem can be solved in a very limited way with the use of OVI. In some advanced analysis the pixels surrounding vortex points are used for reconstructing the geometry of the optical vortex core [18,19], but still most of pixels are useless. There is one more solution to this problem. One can combine the OVI methodology with other techniques. Combining different techniques within a single device is widely used in modern engineering. This is a good method for overcoming various limitations characteristic of a single technique based instrument. In this work we propose to combine the OVI methodology with Fourier interferogram analysis. At the most three

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0030-3992/$ - see front matter
http://dx.doi.org/10.1016/j.optlastec.2012.11.028

different two-beam interferograms $A+B$, $A+C$, $B+C$ can be used in the OVI. To measure directly one set of a two-beam interferogram, the OVI system must be equipped with one shutter. All three sets require three electronically driven shutters to provide sufficient speed [6]. The other solution is to adapt the carrier frequency technique proposed by Takeda [20,21]. In this case we do not alter the OVI optical system. Moreover, no more interferograms are needed apart from the one taken in the standard OVI measurement. It is shown that applying carrier frequency method we can read the phase distribution of the measured wavefront. The same can be done at the vortex points just by using OVI techniques. The OVI measurements are more accurate and can be used for correcting the results obtained with the carrier frequency method. The results corresponding to the three sets of two wave interferograms i.e. $A+B$, $A+C$, $B+C$ can also be compared. One of them $A+B$ bounds the two reference waves. A few ways can be proposed for using this cross information for error corrections but there this shall not be discussed here.

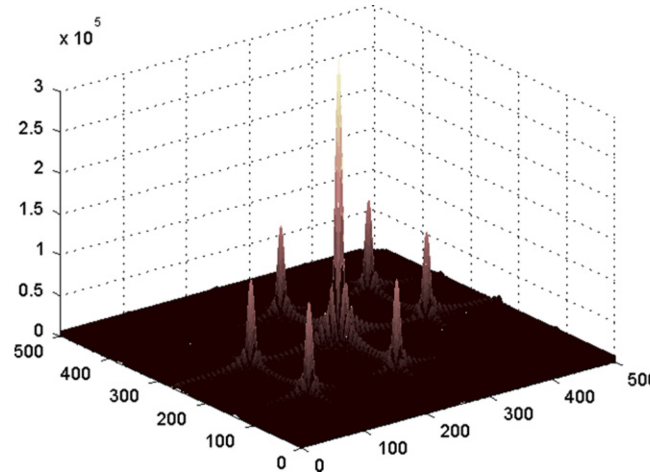

Fig. 1. The exemplary Fourier image of the three beam interferogram.

2. Methodology

Let us consider the interference of the three plane waves $u_A+u_B+u_C$, where

$u_X=A_0e^{ik_xr}$ and $x=A,B,C$

The interference pattern of such an interference field can be described with the equation

$$f(x,y)=a(x,y)+b(x,y)\{\cos 2\pi(\alpha_{AB}x+\beta_{AB}y+\varphi_{AB}) +\cos 2\pi(\alpha_{AC}x+\beta_{AC}y+\varphi_{AC})+\cos 2\pi(\alpha_{BC}x+\beta_{BC}y+\varphi_{BC})\} \quad (1)$$

where α_{ij} and β_{ij} $(i,j=A, B, C)$ are the spatial carrier frequencies in the x and y directions for the pairs of the interferograms. It can be rewritten in the form of

$$f(x,y)=a(x,y)+c_1(x,y)e^{2\pi i(\alpha_{AB}x+\beta_{AB}y)}+c_1^*(x,y)e^{-2\pi i(\alpha_{AB}x+\beta_{AB}y)} +c_2(x,y)e^{2\pi i(\alpha_{AC}x+\beta_{AC}y)}+c_2^*(x,y)e^{-2\pi i(\alpha_{AC}x+\beta_{AC}y)} +c_3(x,y)e^{2\pi i(\alpha_{BC}x+\beta_{BC}y)}+c_3^*(x,y)e^{-2\pi i(\alpha_{BC}x+\beta_{BC}y)} \quad (2)$$

where

$$c_1(x,y)=\tfrac{1}{2}\,b(x,y)e^{i\varphi_{AB}},\quad c_1^*(x,y)=\tfrac{1}{2}b(x,y)e^{-i\varphi_{AB}} \quad (3a)$$

$$c_2(x,y)=\tfrac{1}{2}\,b(x,y)e^{i\varphi_{AC}},\quad c_2^*(x,y)=\tfrac{1}{2}b(x,y)e^{-i\varphi_{AC}} \quad (3b)$$

$$c_3(x,y)=\tfrac{1}{2}\,b(x,y)e^{i\varphi_{BC}},\quad c_3^*(x,y)=\tfrac{1}{2}b(x,y)e^{-i\varphi_{BC}}, \quad (3c)$$

The Fourier transform of Eq. (2) can be written in the form of

$$F(f_x,f_y)=A(f_x,f_y)+C_1(f_x-\alpha_{AB},f_y-\beta_{AB})+C_1^*(f_x+\alpha_{AB},f_y+\beta_{AB}) +C_2(f_x-\alpha_{AC},f_y-\beta_{AC})+C_2^*(f_x+\alpha_{AC},f_y+\beta_{AC}) +C_3(f_x-\alpha_{BC},f_y-\beta_{BC})+C_3^*(f_x+\alpha_{BC},f_y+\beta_{BC}) \quad (4)$$

As it can be concluded from Eq. (4) the Fourier spectrum consists of seven spots (see Fig. 1). The central one corresponds to zero spatial frequency, and the six off-side spots corresponds to the series of two beam interferograms coded in the interference pattern. As a result of placing a wedge into an object arm the carrier frequency and fringe orientation in interferograms AC and BC will change. The change of the carrier frequency from BC to BC' will cause a shift of the spots in the Fourier domain. If the transformed function is rotated, then the coordinate system in the Fourier spectrum is rotated by the same angle. The wedge angle and its orientation can be calculated from the relative spot positions.

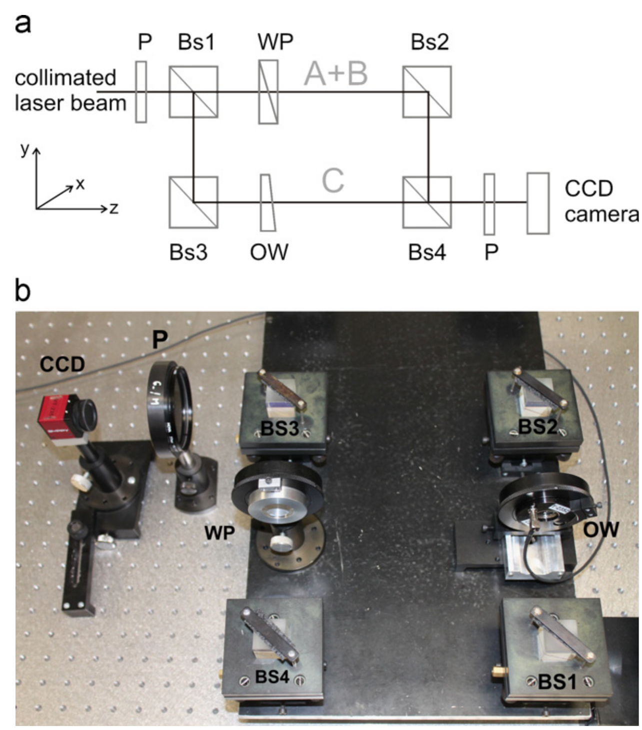

Fig. 2. (a) The optical setup of the three beam interferometer. P—polarizers; Bs1, Bs2, Bs3 and Bs4—beam splitters; WP—Wollaston prism; OW—optical wedge, (b) the photograph of the setup.

An experimental setup of the three-beam interferometer is presented in Fig. 2. For system simplicity we decided to use the OVI version with the Wollaston prism [10]. Four non-polarizing cube beam splitters formed a Mach–Zehnder interferometer. An optical wedge was introduced into the interferometer's object arm. The wave passing through the object arm is denoted as C. The wedge was mounted on the computer controlled rotation stage which enabled its rotation around the optical axis of the interferometer's object arm (z-axis). A collimated beam of coherent laser source (He–Ne laser, 633 nm) was used. The Wollaston prism (made of quarz, the wedge angle 5°) was placed into the reference arm. The Wollaston prism generates two plane waves denoted below as A and B. An example of the output interferogram is shown in Fig. 3(a). The following method for the interferogram analysis is proposed. First the Fourier transform (Fig. 3b) of the image recorded by the CCD camera is calculated. The CCD