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Evolution of hyperbolic-secant optical pulses towards wave breaking in quintic nonlinear fibers

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ABSTRACT

Evolutions of the frequency chirps, shapes, and spectra of initial hyperbolic-secant pulses towards wave breaking are numerically investigated in the normal dispersion regimes of optical fibers with quintic nonlinearity and the developing chirps and the characteristic distances of wave breaking are analytically processed approximately. The results show that quite different mathematical expressions from those of initial Gaussian pulses are obtained. Moreover, the wave breaking here will be more intense for more oscillation peaks will appear in the pulse wings and the breaking process will last longer distance before rectangle-shaped pulses form at last. However, the quintic nonlinearity plays a similar role to the case of initial Gaussian pulses in developing chirps and bringing forward or retarding the wave breaking. The wave breaking distance will also decrease with increase of the quintic nonlinearity and the soliton order.

1. Introduction

As an important and universal phenomenon, optical wave breaking, which results from the interaction between group velocity dispersion (GVD) and self-phase modulation (SPM) but occurs in the normal dispersion regime of fibers [1] or silicon-on-insulator optical waveguides [2], will lead to degradation of pulse quality due to its appearance of oscillations in the wings of the pulse and in the sidelobes on the pulse spectrum. Therefore, this phenomenon is generally thought as a detrimental factor of the maximal and high quality pulse compression [1], wave-breaking-free operation in fiber lasers [3], etc., which should be avoided in practice as far as possible. Of course, the wave breaking characteristics are very sensitive to the initial shape profile of the pulse. Specially, previous studies have already revealed that initially parabolic pulses can obtain self-similar propagation [4]. Obviously, this is very beneficial to high power fiber lasers. Moreover, recent more detailed research concluded quite surprisingly and interestedly that the wave breaking may even turn out to be beneficial in many applications [5]. For all these reasons above, the wave breaking has long been an important topic and has been extensively investigated experimentally [1,3,6–8], numerically [1,4–7,9–13], and approximately analytically [4,10,11,13,14]. However, as far as we know, these studies are nearly limited to the case of Kerr nonlinearity. Many previous researches have revealed that for high incident optical intensities or materials with very high nonlinear coefficients such as semiconductor doped glass fibers, quintic nonlinearity will take effect and influence considerably the optical soliton propagation [15], modulation instability [16], and optical wave breaking [17]. In Ref. [17], the wave breaking of the initially chirp-free Gaussian (ICFG) pulse has been numerically and approximately analyzed in an optical fiber with quintic nonlinearity. In this paper, studies on the optical wave breaking in case of quintic nonlinearity will be extended to the initially chirp-free hyperbolic-secant (ICFHS) pulse.

2. Theoretical analysis

The slowly varying envelopes ψ of the optical field should satisfy the following extended nonlinear Schrödinger equations with quintic nonlinearity [15]:

$$i\frac{\partial\psi}{\partial z}-\frac{1}{2}\beta_2\frac{\partial^2\psi}{\partial T^2}+\gamma_1|\psi|^2\psi+\gamma_2|\psi|^4\psi=0 \quad (1)$$

where β_2, γ_1, γ_2, z, and T are the second-order group velocity dispersion (GVD) coefficient, cubic nonlinear coefficient, quintic nonlinear coefficient, propagating distance, and the retarded time, respectively. The last two terms on the left-handed side of Eq. (1) stand for self-phase modulation (SPM), which contains contributions of cubic and quintic nonlinearity.

In optical wave breaking and even other optical propagation processes, the developed frequency chirps of pulses undoubtedly

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play an important role. Therefore, we intend to analyze the frequency chirps developed by GVD and SPM effects in advance to understand the underlying qualitative mechanisms of the optical wave breaking. To do this, we adopt the similar approach of Refs. [13] and [17]. By introducing the real amplitude and phase of $\psi(z,T)$ according to $\psi(z,T)=A(z,T)\exp[i\phi(z,T)]$ and separating Eq. (1) into real and imaginary parts, one can obtain the following equations:

$$\phi_z=-\frac{\beta_2}{2}\frac{A_{TT}}{A}+\frac{\beta_2}{2}\omega_c^2+\gamma_1A^2+\gamma_2A^4 \quad (2)$$

$$(A^2)_z=-\beta_2(A^2\omega_c)_T \quad (3)$$

where the frequency chirp is defined as $\omega_c=-\partial\phi/\partial T$. Eq. (3) indicates that the pulse shape evolution is closely related to the chirp variation. According to Eq. (2), the initial evolution of the phase should be of the following form:

$$\phi(z,T)=\phi(0,T)+\left(-\frac{\beta_2}{2}\frac{A_{TT}}{A}+\frac{\beta_2}{2}\omega_c^2+\gamma_1A^2+\gamma_2A^4\right)\Big|_{z=0}z \quad (4)$$

and the corresponding developed chirp can be obtained as

$$\omega_c(z,T)=\omega_c(0,T)+\left[-\frac{\beta_2}{2}\omega_c^2(0,T)+\frac{\beta_2}{2}\frac{A_{TT}(0,T)}{A(0,T)}-\gamma_1A^2(0,T)-\gamma_2A^4(0,T)\right]\Big|_T z \quad (5)$$

For an ICFHS pulse of the form $\psi(0,T)=A_0\mathrm{sec}h(T/T_0)$, Eq. (5) yields

$$\omega_c(z,T)\cong-\frac{\beta_2z}{T_0^3}\left[Q_1-2+Q_2\mathrm{sec}h^2\left(\frac{T}{T_0}\right)\right]\tanh\left(\frac{T}{T_0}\right)\mathrm{sec}h^2\left(\frac{T}{T_0}\right) \quad (6)$$

where $Q_1=-(2\gamma_1/\beta_2)T_0^2A_0^2$ and $Q_2=-(4\gamma_2/\beta_2)T_0^2A_0^4$. Obviously, if no quintic nonlinearity exists, i.e., $\gamma_2=0$ and then $Q_2=0$, Eq. (6) will accord with the case of cubic nonlinearity [13].

In the normal dispersion region where the wave breaking occurs, the parameters satisfy $\beta_2>0$, $\gamma_1>0$, and then $Q_1<0$. However, Q_2 may be >0, <0, or $=0$ depending on the sign of γ_2. From Eq. (6), one can easily see that the positive ($\gamma_2>0$) and negative quintic nonlinearity ($\gamma_2<0$) can, respectively, increase and decrease the chirp amount. Furthermore, one can still realize that the chirp given by Eq. (6) is always nonmonotonic, even in the linear and cubic nonlinearity case. Accordingly, as Ref. [13] pointed out, a nonmonotonic chirp implies that there exists overtaking of different parts of the pulse and wave breaking ultimately occurs. The wave-breaking distance should take the following form [13]:

$$z_{wb}=-1/(\beta_2d\omega_c/dT) \quad (7)$$

where, the relation $d\omega_c/dT<0$ must hold because the parameter z_{wb} must take the positive value. According to Eqs. (6) and (7), the normalized wave-breaking distance Z_{wb} can be deduced as

$$Z_{wb}^2=-\frac{4}{\pi^2Y(\mathrm{sec}h^2\tau)} \quad (8)$$

where, $Z=z/z_0$ is the normalized distance, $z_0=\pi L_D/2=\pi T_0^2/(2\beta_2)$ is the soliton period, L_D is the dispersion length of the fiber, $\tau=T/T_0$ is the normalized time coordinate,$Y(x)=[-5Q_2x^2+(4Q_2-3Q_1+6)x+2(Q_1-2)]x$, and $x=\mathrm{sec}h^2\tau$. When the right-handed side of Eq. (8) is positive, the wave-breaking occurs and the corresponding minimum value of Z_{wb} is the critical normalized distance or characteristic distance where the wave-breaking first begins. One can deduce that, the relation $[4-3P]^2+30\quad P>0$ always holds where $Q_2\neq0$ and $P=(Q_1-2)/Q_2$. As described above, when $Y(x)$ is negative and takes the minimal value, the wave-breaking first begins. Thus, the corresponding critical normalized distance Z_{wbc} or the threshold condition can be deduced as

$$Z_{wbc}=\begin{cases}-\frac{2}{\pi Y(x_1)} & (Q_1<2,Q_2<0)\\ -\frac{2}{\pi Y(x_2)} & (Q_1<2,Q_2>0)\end{cases} \quad (9)$$

where the parameters x_j ($j=1$, 2) are, respectively, of the following forms:

$$x_1=\frac{2(4-3P)+\sqrt{[2(4-3P)]^2+120P}}{30} \quad (10)$$

$$x_2=\frac{2(4-3P)-\sqrt{[2(4-3P)]^2+120P}}{30} \quad (11)$$

Adopting the similar analysis here, one can deduce that the critical normalized distance Z_{wbc} or the threshold condition for is

$$Z_{wbc}=\frac{2}{\pi}\sqrt{\frac{3}{2-Q_1}}\quad(Q_1<2)$$

when there are no quintic nonlinearities ($Q_2=0$), which accords with that of Ref. [13].

Obviously, the approximate analytical expressions above are mathematically quite different from those of initial Gaussian pulses [17].

3. Calculations and discussions

To numerically simulate Eq. (1) conveniently, one usually adopts the following normalized form of Eq. (1):

$$i\frac{\partial u}{\partial\xi}-\frac{1}{2}\mathrm{sgn}(\beta_2)\frac{\partial^2u}{\partial\tau^2}+|u|^2u+\frac{rr}{N^2}|u|^4u=0 \quad (12)$$

where u, ξ, N, and rr are normalized envelope of the optical field, normalized distance, soliton order, and quintic nonlinearity related parameter, respectively, [17] and sgn stands for the symbol function.

To compare the wave breaking characteristics of ICFHS pulses with those of ICFG pulses, we set the same parameters of rr and N to those of ICFG pulses [17] in the following numerical calculations. One may naturally infer that their wave breaking characteristics will be similar to some extent for their similar initial shapes. The numerically calculated shape evolutions are shown in Fig. 1. Just as one expects, the quintic nonlinearity plays a similar role to the case of ICFG pulses in bringing forward or retarding the wave breaking. However, the difference is that the wave breaking here will be more intense for more oscillation peaks will appear in the pulse wings. Further investigations indicate that the wave breaking process will last longer distance before rectangle-shaped pulses form at last. Moreover, wider temporal width of the pulse can be observed obviously when the other parameters are the same.

The approximate analytical descriptions for the critical wave breaking distance can be obtained by taking the minimum value of Z_{wb} in Eq. (8) and are shown in Fig. 2. Being similar to the case of ICFG pulses, the wave breaking distance will also decrease with increase of the quintic nonlinearity and the soliton order. The difference is that, however, the characteristic distance of wave breaking here is longer than that of the ICFG pulse. In addition, the numerical simulations for the wave breaking distance have also been provided in Fig. 2 to support our analytical results. Obviously, the two results accords well with each other except for some minor deviations between them. One can see that the numerical wave breaking distance is a little shorter than the