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Bias-corrected estimation for speculative bubbles in stock prices☆

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ABSTRACT

We provide a comparison of different finite-sample bias-correction methods for possibly explosive autoregressive processes. We compare the empirical performance of the downward-biased standard OLS estimator with an OLS and a Cauchy estimator, both based on recursive demeaning, as well as a second-differencing estimator. In addition, we consider three different approaches for bias-correction for the OLS estimator: (i) bootstrap, (ii) jackknife and (iii) indirect inference. The estimators are evaluated in terms of bias and root mean squared errors (RMSE) in a variety of practically relevant settings. Our findings suggest that the indirect inference method clearly performs best in terms of RMSE for all considered levels of persistence. In terms of bias-correction, the jackknife works best for stationary and unit root processes, but with a typically large variance. For the explosive case, the indirect inference method is recommended. As an empirical illustration, we reconsider the "dot-com bubble" in the NASDAQ index and explore the usefulness of the indirect inference estimator in terms of testing, date stamping and calculations on overvaluation.

1. Introduction

Estimating the persistence of financial and economic time series is a long standing issue in applied econometrics. The most common framework for assessing the persistence is the autoregressive model. A major practical problem is the inherent downward bias of the conventional OLS estimator. Its bias increases along two dimensions: a small sample size and a true autoregressive parameter in the vicinity of unity are disadvantageous. Given a relatively small sample size arising from e.g. sample splitting, rolling windows, low frequency or data availability, it is a complicated task to estimate the persistence if the process (i) is either stationary, but highly persistent, (ii) exhibits a unit root or (iii) behaves explosively. The bias function is highly nonlinear and changes its derivative in the local-to-unity region.

In finance and economics, it is a well established fact that most time series are characterized by high persistence and stochastic trends, see e.g. Nelson and Plosser (1982) and Schotman and van Dijk (1991). During periods of bubbles (or crises) some financial and economic time series are likely to exhibit even explosive behavior. Recent examples for time series with temporary explosive roots are stock prices (Phillips et al., 2011), house prices (Caspi, 2016; Engsted et al., 2016; Shi, 2017), exchange rates (Steenkamp, 2018; Hu and Oxley, 2018) and commodity prices (Gutierrez, 2013; Etienne et al., 2014; Figuerola-Ferretti et al., 2015). Moreover, there is also evidence in art markets (Kräussl et al., 2016; Assaf, 2017) and target balances

☆ This paper is a substantial revision of Kaufmann and Kruse (2013). The working paper by Kaufmann and Kruse (2013) includes additional simulation results with a broader scope. When pointing out further unreported results in Section 3, we refer to their results to converse space and to streamline the discussion. The authors gratefully acknowledge the constructive comments and suggestions made by an anonymous referee. We are grateful to the participants of the 4th Amsterdam-Bonn Workshop in Econometrics, the 21st Symposium of the Society for Nonlinear Dynamics and Econometrics in Milan, the 7th Nordic Econometric Meeting in Bergen, the 2nd Time Series Workshop in Rimini and the 10th BMRC-QASS conference in London for most inspiring discussions and helpful comments. We would like to thank in particular Christoph Hanck, Karim Abadir, Jörg Breitung, Simon Burke, Matei Demetrescu, Rod McCrorie, Jun Ma, Hashem Pesaran and Philipp Sibbertsen for helpful remarks. Robinson Kruse gratefully acknowledges support from CREATES - Center for Research in Econometric Analysis of Time Series (DNRF78), funded by the Danish National Research Foundation.

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(Potrafke and Reischmann, 2014).

This work compares several bias-correction estimators and techniques by means of a Monte Carlo study. Among these are the (i) Cauchy estimator by So and Shin (1999a) which builds upon another (ii) recursive mean adjusted estimator, see So and Shin (1999b). The Cauchy estimator is approximately median-unbiased for unit root and explosive processes. This property makes it attractive in comparison to the classic analytic median bias-corrections proposed in Andrews (1993), Andrews and Chen (1994) and Roy and Fuller (2001) which rule out explosiveness by construction. We also consider a recently proposed (iii) estimator based on second differencing by Chen and Kuo (2013). In addition, we consider the (iv) bootstrap, (v) jackknife and (vi) indirect inference approaches. In more detail, we study the bootstrap-based bias-correction procedure by Kim (2003) which builds on Kilian (1998). The jackknife correction (based on Efron, 1979) is recently studied in Chambers (2013), and the indirect inference estimator (see e.g. MacKinnon and Smith, 1998; Gouriéroux et al., 2000) is suggested by Phillips et al. (2011).

While the main body of the literature focusses on stationary autoregressive models and on the unit root case, mildly explosive behavior has received much less attention. As a consequence, the finite-sample properties of recently suggested techniques are only partly explored and a comprehensive comparison with a focus on explosive roots has not been conducted yet. Due to the nonlinearity in the bias function, it is unclear whether existing recommendations for the stationary and unit root case carry over to the situation with explosive roots. Even some bias-corrected estimators exclude explosive behavior by construction. For a practitioner it is important to know which estimator (if any) performs best overall, i.e. for stationary, unit root and explosive processes. This paper intends to fill these gaps and to provide recommendations for practical applications.

In our Monte Carlo study, we evaluate the performance of the estimators by means of bias and root mean squared errors (RMSE) in small samples. We find that the indirect inference estimator performs very well in terms of RMSE and routinely outperforms its competitors. The recommendations for bias-correction (without considering the variance of estimators) are more diverse. We distinguish situations where the practitioner (i) aims at using a robust method with balanced performance across the levels of persistence or (ii) can either rule out stationarity or explosiveness a priori. Regarding (i), indirect inference and bootstrap are the best robust choices in terms of bias-reduction. In case (ii) the jackknife is highly recommendable in absence of explosiveness, while the bootstrap and indirect inference perform very well for explosive series. Interestingly, these two methods rank second and third for stationary and unit root series which makes their use advisable in case (i). It is worthwhile to emphasize that the indirect inference estimator is the clear winner in terms of RMSE overall.

We provide an empirical illustration to the real NASDAQ composite price index between 1973:01 and 2005:06. This time span should cover explosive regimes due to the presence of the "dot-com bubble". The data has been intensively studied in the academic literature (see, for example Phillips et al., 2011; Homm and Breitung, 2012; Harvey et al., 2017) and facilitates to emphasize the relevance of precise persistence estimation. For ease of comparison, we consider the OLS and indirect inference estimator in a rolling window fashion in order to test for speculative bubbles, date stamp the start and the end of the "dot-com bubble" and to estimate the bubble growth rate.

The paper is organized as follows. Section 2 describes the different estimators and bias-correction techniques. We present our simulation results in Section 3. The empirical illustration is given in Section 4. Conclusions are drawn in Section 5.

2. Finite-sample bias-corrections

2.1. Bias of the OLS estimator

The inherent bias of the OLS estimator in autoregressive models is our point of departure. The complicated estimation of autoregressive processes in finite samples sparked a fruitful area of research, see e.g. Kiviet and Phillips (2012) for a recent survey on the vast literature.[1] We focus on the possibility of mild explosive behavior in a simple and widely applied autoregressive framework with unknown mean. For ease of presentation, we consider a simple first-order model, but our discussion extends to higher-order models in a straightforward way. It is given by

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t \,, \tag{1}$$

where ε_t is a zero-mean white noise process with variance σ.[2] We consider the cases of stationarity and unit roots, i.e. $|\rho| < 1$ and $\rho = 1$, and the case where ρ satisfies $\rho = 1 + c/T$, with $c > 0$ and T being the sample size. In the latter case, the autoregressive parameter is local-to-unity in the sense that $\rho \to 1$ as $T \to \infty$. For finite T, ρ deviates moderately from unity.[2]

It is well known that the OLS bias in the given AR(1) model depends on the true value of the autoregressive parameter ρ and the sample size T. For instance, Bao (2007) uses Nagar-type expansions to show that the expected value of the OLS estimator $\hat{\rho}$ has the form[3]

$$E\left(\hat{\rho}\right) = \rho - \frac{1+3\rho}{T} + \frac{1}{T^2}\left[\frac{3\rho - 9\rho^2 - 1}{1-\rho} + \frac{\mu^2}{\sigma^2}\frac{1+3\rho}{(1-\rho)^2}\right] + o\left(T^{-2}\right) \,.$$

The smaller the sample size, the more severe is the downward bias. For fixed T, the downward bias is strongest when $\rho \to 1$. To the best of our knowledge, no analogous approximations have been yet derived for the explosive case with unknown mean. An exception is Phillips (2012), where the author shows that the bias behaves as $O(\rho^{-T})$ for the special case when $\mu = 0$ and fixed $\rho > 1$.[4] As we are mainly interested in the practically more relevant case with an unknown mean, we focus on numerical bias-correction methods in the following. However, the analytic bias result for the explosive case with zero intercept already provides several interest insights: the bias function is highly nonlinear and changes its derivative in the local-to-unity region, see also MacKinnon and Smith (1998) for early experimental evidence which we replicate by some introductory simulations in Fig. 1.

Fig. 1 shows the bias of the OLS estimator $\hat{\rho}$ for ρ in $y_t = \mu + \rho y_{t-1} + \varepsilon_t$ for two different sample sizes of $T = 25$ and $T = 50$.[5] It can be seen that the bias reduces much quicker for explosive processes as ρ increases. It approaches zero for large (in comparison to the sample size) values of ρ. But, the estimation of mildly explosive processes with roots near unity is still heavily biased. The problem persists for higher-order autoregressive models for which analytic bias formulas depend on the autoregressive lag order.

[1] Kendall (1954), Shaman and Stine (1988), Tjøstheim and Paulsen (1983), Tanaka (1984) and Abadir (1993) provide analytic derivations of asymptotic expansions which can be used for bias-correction, see also Abadir (1995) for the context of unit root testing. Bao and Ullah (2007) provide general results on the second-order bias and the mean squared error.

[2] Asymptotic theory for mildly explosive autoregressions is developed in Phillips and Magdalinos (2007). Wang and Yu (2015) provide asymptotic results for explosive autoregressions.

[3] For the following result, normality is assumed in addition to $\rho < 1$ and $y_0 = 0$. More detailed expressions can be found in Bao (2007) for the case of non-normality and non-zero initialization. Stationarity is, however, required for such Nagar-type expansions.

[4] An analogous (and more complicated) formula for the mildly explosive case with $\rho = 1 + c/T$, $c > 0$ is also provided.

[5] The true autoregressive parameter ρ (on the x-axis) ranges from 0.6 to 1.2. For simplicity, we set $\mu = 0$, $y_0 = 0$ and $\varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$.