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A term structure model under cyclical fluctuations in interest rates☆

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ABSTRACT

We propose a flexible yet tractable model of the term structure of interest rates (TSIR). Term structure models attempt to explain how interest rates depend on their maturities at a given point in time, characterizing the relationship between short-term and long-term rates. Our model can reproduce and fit a variety of TSIR shapes by capturing cyclical fluctuations of interest rates, different monetary policy reactions as witnessed pre- and post-crisis as well as the effect of the business cycle or exogenous shocks. Our modelling approach also provides a characterization of long-term fluctuations in the mean level of interest rates unveiling the effects of monetary policy interventions in interest rates. Furthermore, using daily US data, we compare the empirical ability of our model to both fit and forecast the TSIR under different economic scenarios. We show that our model improves pricing and risk management by fitting and predicting interest rates more accurately and precisely than do existing TSIR models.

1. Introduction

Zero-coupon interest rates at very short maturities are determined by the implementation of monetary policy. However, longer term zero-coupon interest rates are the drivers relevant for private sector decisions since the cost of the credit needed to finance durable consumption purchases and investment goods is usually related to the interest rates at longer maturities, such as 10-year rates. The reference to zero-coupon rates reflects the fact that these specify the required payment on the return of an amount of money loaned over a given period of time, the maturity of the interest rate. The price of a bond or any other financial asset involving a flow of payments can be easily determined using zero-coupon rates at the maturities at which a given flow will take place in the future.

The so-called term structure of interest rates (TSIR) is the representation of zero-coupon interest rates as a function of the time to maturity. It codifies the relationship between short-term rates and medium-term and long-term rates. The TSIR is an essential input for the analysis of fixed-income markets as well as for monetary policy design and implementation, and, over the years, many models have been proposed to describe the nature of the relationships between zero-coupon interest rates at different maturities. This amounts to estimating how monetary policy interventions affect the costs of consumption and investment decisions and how they potentially influence the business cycle in a given economy.

TSIR models can be endogenous or exogenous. Endogenous models consider changes in interest rates at different maturities as being determined by one or more common factors for which they assume a specific stochastic behavior. Under such assumptions the current TSIR can be derived from the model. See, for instance, Vasicek (1977), Cox et al. (1985), Duffie and Kan (1996), Moreno and Platania (2015), or Renne (2016), among others.

In contrast, exogenous models consider the current TSIR as an input and derive the future changes in interest rates that prevent intertemporal arbitrage opportunities, i.e. profitable trading strategies that could be implemented by combining available fixed income assets with dif-

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ferent maturities. Some examples are Hull and White (1990, 1993) and Heath et al. (1992). For detailed reviews on TSIR models see James and Webber (2001), Brigo and Mercurio (2006), Nawalkha et al. (2007), Filipović (2009), Andersen and Piterbarg (2010), and Munk (2015), among others.

We work in a standard framework of one-factor continuous-time endogenous models for the TSIR with the instantaneous spot rate as its single factor. In particular, we regard the instantaneous zero-coupon interest rate as following an Ornstein-Uhlenbeck stochastic process. Consequently, its time evolution is governed by the tendency to revert to the long-term mean while reacting to shocks from a fundamental Wiener process describing interest rate innovations.

What is new in our proposal is the assumption that the long-term mean of the instantaneous zero-coupon interest rate displays cyclical behavior. This cyclical behavior is modelled through a Fourier series. This specification allows us to simulate the wide variety of shapes actually observed for the TSIR. The cyclical behavior in the long-term mean may arise because of reactions to monetary policy interventions intended to influence the business cycle.

Indeed, the relationship between the term structure and macroeconomic variables has been widely documented in the literature (see Diebold et al. (2006), Evans and Marshall (2007), Tillmann (2007), and Paccagnini (2016), among many others). Our specification is in the spirit of the Hull and White (1994) term structure model in that it considers time varying parameters. We go one step further in dealing with seasonality explicitly as well as in allowing for a specific dependence of interest rates on the business cycle. Even though we use it in this paper to estimate the daily term structure of interest rates, our modelling strategy could also be used to estimate the time variation in the unobserved mean level in interest rates. Among other things, that would allow for evaluating the relevance of monetary policy in interest rate determination, as it is done in Mallick et al. (2017).

Our proposal is an alternative to models usually adopted by central banks, such as Nelson and Siegel (1987) and Svensson (1994), that achieve this kind of flexibility by using exponential functions while obtaining a good fit to market interest rates. In these models, the instantaneous interest rate expected at any future time is described by a combination of exponential functions of the given time horizon. Its use as the single factor of the TSIR means that interest rates at any other maturity are also expressed by combinations of exponential functions of their maturities.

Similarly, the Ornstein-Uhlenbeck specification for the instantaneous rate means that any other interest rate is expressed as a nonlinear function of its maturity. Moreover, in our model, interest rates follow a Gaussian distribution,[1] a relevant issue given the theoretical zero bound on interest rates and the negative interest rates that have been observed recently in some countries.[2]

Estimating the parameters in the representations for interest rates at the different maturities we can recover values for parameters in the process for the instantaneous interest rate as well as for parameters in the Fourier series describing the cyclical behavior of the mean reversion level. We analyze the in-sample and out-of-sample performance of this model to explain the TSIR, using Vasicek (1977) and Nelson and Siegel (1987) as benchmark models.

Vasicek (1977) was one of the first TSIR models and it has inspired a number of interesting extensions. Its main assumptions are that interest rates converge to a (constant) long-term mean and that the volatility of changes in interest rates is constant. Consequently, interest rates follow a Gaussian distribution. As our model generalizes Vasicek (1977), the Vasicek model is a natural benchmark with which to assess the gains attained by our modelling of the mean level of interest rates and the relevance of its (potential) flexibility.

On the other hand, the Nelson and Siegel (1987) model is a significant benchmark since it is one of the most popular models of the yield curve and is used by many central banks in the implementation and evaluation of monetary policy. Moreover, Diebold and Li (2006) showed the good forecasting performance of this model in comparison with ten alternative competitors, including Fama and Bliss (1987) and Cochrane and Piazzesi (2005).

With respect to our data set, our model outperforms its competitors both in-sample and out-of-sample. It provides a more precise fit to actual market values as well as better forecasts.

The paper is organized as follows. Section 2 introduces the analytical model and characterizes the TSIR. Section 3 presents the in-sample and out-of-sample empirical analyses of the model. Finally, Section 4 summarizes the main findings and conclusions.

2. The term structure model

In this section we introduce the model, present the partial differential equation that must be satisfied by the price of any derivative asset, obtain bond pricing equations, and characterize the TSIR.

2.1. The model

Let r_t denote the instantaneous interest rate at time t. We assume that the time evolution of r_t is given by an Ornstein-Uhlenbeck process, defined by the stochastic differential equation

$$dr_t = \kappa(f(t) - r_t)dt + \sigma dW_t, \tag{1}$$

where $\kappa, \sigma \in \mathbb{R}^+$ and W_t is a standard Wiener process. In addition, we assume that the mean-reversion level, $f(t)$, follows a time-dependent process driven by a Fourier series,

$$f(t) = \sum_{n=0}^{\infty} \mathrm{Re}\left[A_n e^{in\omega t}\right],$$

where we only consider the real part of the Fourier series since it is the only part that makes economic sense. Note that $A_n \in \mathbb{C}$ for all n, so there is a phase factor contained in A_n. In more detail, $A_n = A_{n,x} + iA_{n,y}$ where $A_{n,x}, A_{n,y} \in \mathbb{R}$. Hence, $A_{n,x}$ and $A_{n,y}$ denote, respectively, the amplitude and phase of the fluctuations of the instantaneous rate. Finally, this model specializes to that in Vasicek (1977) by taking $A_n = 0$ for $n \in \mathbb{N} - \{0\}$.

Now, let $\Lambda(r_t, t)$ denote the market price of risk, which is assumed to be constant, $\Lambda(r_t, t) = \lambda$. Then the risk-neutral version of the process (1) is given by

$$dr_t = \mu_r dt + \sigma d\widetilde{W}_t, \tag{2}$$

for

$$\mu_r = \kappa\left(\alpha + g(t) - r_t\right),$$

$$\alpha = A_0 - \frac{\lambda\sigma}{\kappa},$$

$$g(t) = \sum_{n=1}^{\infty} \mathrm{Re}\left[A_n e^{in\omega t}\right] = f(t) - A_0, \tag{3}$$

where $A_0 \in \mathbb{R}$ and $\widetilde{W}_t = W_t + \lambda t$ is a standard Wiener process under the risk-neutral measure $\widetilde{P}$. The following proposition describes the solution of the stochastic differential equation (2).

[1] In usual market conditions such a distribution is considered a disadvantage, and this has often motivated the use of CIR-type models, which impose the positivity of interest rates. We strongly thank one of the referees for focusing our attention on this issue.

[2] For a deep analysis of Gaussian TSIR models see Realdon (2016) and references therein.