Economic Modelling xxx (2017) 1–7

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.journals.elsevier.com/economic-modelling

Testing normality for unconditionally heteroscedastic macroeconomic variables

Hamdi Raïssi

Instituto de Estadística, PUCV, Av. Errazuriz 2734, Valparaíso, V región, Chile

ARTICLE INFO

JEL:
C12
C15
C18

Keywords:
Unconditionally heteroscedastic time series
Jarque-Bera test

ABSTRACT

In this paper the testing of normality for unconditionally heteroscedastic macroeconomic time series is studied. It is underlined that the classical Jarque-Bera test (JB hereafter) for normality is inadequate in our framework. On the other hand it is found that the approach which consists in correcting the heteroscedasticity by kernel smoothing for testing normality is justified asymptotically. Nevertheless it appears from Monte Carlo experiments that such a methodology can noticeably suffer from size distortion for samples that are typical for macroeconomic variables. As a consequence a parametric bootstrap methodology for correcting the problem is proposed. The innovations distribution of a set of inflation measures for the U.S., Korea and Australia are analyzed.

1. Introduction

In the econometric literature, the Jarque and Bera (1980) test is routinely used to assess the normality of variables. The properties of this test are well documented for stationary conditionally heteroscedastic processes. For instance Fiorentini et al. (2003), Lee et al. (2010, 2012) investigated the JB test in the context of GARCH models. However few studies are available on the distributional specification of time series in presence of unconditional heteroscedasticity. Drees and Stărică (2002), Mikosch and Stărică (2004) and Fryźlewicz (2005) investigated the possibility of modeling financial returns by nonparametric methods. To this end, Drees and Stărică (2002) and Mikosch and Stărică (2004) examined the distribution of S&P500 returns corrected from heteroscedasticity. On the other hand Fryźlewicz (2005) pointed out that large sample kurtosis for financial time series may be explained by non constant unconditional variance. In general we did not found references that specifically address the problem of assessing the distribution of unconditionally heteroscedastic time series. Note that non constant variance constitutes an important pattern for time series in general, and macroeconomic variables in particular. Reference can be made to Sensier and van Dijk (2004) who found that most of the 214 U.S. macroeconomic time series they studied have a time-varying variance. In this paper we aim to provide a reliable methodology for testing normality for small samples time series with non constant unconditional variance.

The structure of the paper is as follows. In Section 2 we first set the dynamics ruling the observed process. In particular the unconditional heteroscedastic structure of the errors is given. The inadequacy of the standard JB test in our framework is highlighted. The approach consisting in correcting the errors from the heteroscedasticity for building a JB test is presented. We then introduce a parametric bootstrap procedure that is intended to improve the normality testing for unconditionally heteroscedastic macroeconomic time series. In Section 3 numerical experiments are conducted to shed some light on the finite sample behaviors of the studied tests. In particular it is found that when estimating the non constant variance structure by kernel smoothing, a correct bandwidth choice is a necessary condition for the good implementation of the normality tests based on heteroscedasticity correction. We illustrate our outputs by examining the distributional properties of the U.S., Korean and Australian GDP implicit price deflators.

2. Testing normality in presence of unconditional heteroscedasticity

We consider processes (y_t) which can be written as

$$y_t = \omega_0 + x_t,$$

$$x_t = \sum_{i=1}^{p} a_{0i} x_{t-i} + u_t, \tag{2.1}$$

where $y_1, \dots, y_n$ are available, n is the sample size and $E(x_t) = 0$. The conditional mean of x_t is driven by the autoregressive parameters $\theta_0 = (a_{01}, \dots, a_{0p})'$, which fulfill the following condition.

E-mail address: hamdi.raissi@pucv.cl.

https://doi.org/10.1016/j.econmod.2017.10.015
Received 29 June 2017; Received in revised form 10 October 2017; Accepted 27 October 2017
Available online XXX

Assumption A0. The $a_{0i} \in \mathbb{R}$, $1 \le i \le p$, are such that $\det(a(z)) \neq 0$ for all $|z| \le 1$, with $a(z) = 1 - \sum_{i=1}^{p} a_{0i} z^i$.

In the assumption A1 below, the well known rescaling device introduced by Dahlhaus (1997) is used to specify the errors process (u_t). For a random variable v we define $\|v\|_q = (E|v|^q)^{1/q}$, with $E|v|^q < \infty$ and $q \ge 1$.

Assumption A1. We assume that $u_t = h_t \epsilon_t$ where:

(i) $h_t \ge c > 0$ for some constant $c > 0$, and satisfies $h_t = g(t/n)$, where $g(r)$ is a measurable deterministic function on the interval $(0, 1]$, such that $\sup_{r\in(0,1]}|g(r)| < \infty$. The function $g(.)$ satisfies a Lipschitz condition piecewise on a finite number of some subintervals that partition $(0, 1]$.

(ii) The process (ϵ_t) is iid and such that $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = 1$, and $E(\|\epsilon_t\|^{8\nu}) < \infty$ for some $\nu > 1$.

The non constant variance induced by A1**(i)** allows for a wide range of non stationarity patterns commonly faced in practice, as for instance abrupt shifts, smooth changes or cyclical behaviors. Note that in the zero mean AR case, tools needed to carry out the Box and Jenkins specification-estimation-validation modeling cycle, are provided in Patilea and Raïssi (2012, 2013) and Raïssi (2015). For $\omega_0 \neq 0$ define the estimator $\hat{\omega} = n^{-1}\sum_{t=1}^{n} y_t$, and $x_t(\omega) = y_t - \omega$ for any $\omega \in \mathbb{R}$. Writing $\hat{\omega} - \omega_0 = n^{-1}\sum_{t=1}^{n} x_t$, it can be shown that

$$\sqrt{n}(\hat{\omega} - \omega_0) = O_p(1), \tag{2.2}$$

using the Beveridge-Nelson decomposition. Now let

$$\hat{\theta}(\omega) = \left(\Sigma_{\underline{x}}(\omega)\right)^{-1} \Sigma_x(\omega), \tag{2.3}$$

where

$$\Sigma_{\underline{x}}(\omega) = n^{-1}\sum_{t=1}^{n} \underline{x}_{t-1}(\omega)\underline{x}_{t-1}(\omega)' \quad \text{and}$$

$$\Sigma_x(\omega) = n^{-1}\sum_{t=1}^{n} \underline{x}_{t-1}(\omega) x_{t-1}(\omega),$$

with $\underline{x}_{t-1}(\omega) = (x_{t-1}(\omega), \ldots, x_{t-p}(\omega))'$. With these notations define the OLS estimator $\hat{\theta}(\hat{\omega})$ and the unfeasible estimator $\hat{\theta}(\omega_0)$. Straightforward computations give $\sqrt{n}(\hat{\theta}(\hat{\omega}) - \hat{\theta}(\omega_0)) = o_p(1)$, so that using the results of Patilea and Raïssi (2012) we have

$$\sqrt{n}(\hat{\theta}(\hat{\omega}) - \theta_0) = O_p(1). \tag{2.4}$$

Once the conditional mean is filtered in accordance to (2.2) and (2.4), we can proceed to the test of the following hypotheses:

$$H_0 : \epsilon_t \sim \mathcal{N}(0, 1) \quad \text{vs.} \quad H_1 : \epsilon_t \text{ has a different distribution},$$

with the usual slight abuse of interpretation inherent of the use JB test for normality testing. Clearly the skewness and kurtosis of u_t correspond to those of ϵ_t. However in practice $E(u_t^3) = 0$ and $E(u_t^4)/E(u_t^2)^2 = 3$ is checked using the JB test statistic:

$$Q_{JB}^{u} = n\left[Q_{JB}^{S,u} + Q_{JB}^{K,u}\right], \tag{2.5}$$

where

$$Q_{JB}^{S,u} = \frac{\hat{\mu}_3^2}{6\hat{\mu}_2^3} \quad \text{and} \quad Q_{JB}^{K,u} = \frac{1}{24}\left(\frac{\hat{\mu}_4}{\hat{\mu}_2^2} - 3\right)^2,$$

with $\hat{\mu}_j = n^{-1}\sum_{t=1}^{n}(\hat{u}_t - \overline{\hat{u}})^j$ and $\overline{\hat{u}} = n^{-1}\sum_{t=1}^{n}\hat{u}_t$. The $\hat{u}_t$'s are the residuals obtained from the estimation step. Let us denote by $\Rightarrow$ the convergence in distribution. If we suppose the process (u_t) homoscedastic ($g(.)$ is constant), then the standard result $Q_{JB}^{u} \Rightarrow \chi_2^2$ is retrieved (see Yu (2007), Section 2.2). However under A0 and A1 with $g(.)$ non constant (the unconditionally heteroscedastic case) we have:

$$Q_{JB}^{K,u} = \frac{1}{24}\left[\kappa_2\left(E\left(\epsilon_t^4\right) - 3\right) + 3\left(\kappa_2 - 1\right)\right] + o_p(1), \tag{2.6}$$

where $\kappa_2 = \frac{\int_0^1 g^4(r)dr}{\left(\int_0^1 g^2(r)dr\right)^2}$. Hence if we suppose the errors process unconditionally heteroscedastic with $E(\epsilon_t^4) = 3$, we obtain $Q_{JB}^{u} = Cn + o_p(n)$ for some strictly positive constant C. As a consequence, the classical JB test will tend to detect spuriously departures from the null hypothesis of a normal distribution in our framework. This argument was used by Fryźlewicz (2005) to underline that unconditionally heteroscedastic specifications can cover financial time series that typically exhibit an excess of kurtosis.

In order to assess the distribution of S&P500 returns, Drees and Stărică (2002) considered data corrected from heteroscedasticity, using a kernel estimator of the variance. We will follow this approach in the sequel considering

$$\hat{h}_t^2 = \sum_{i=1}^{n} w_{ti}(\hat{u}_i - \overline{\hat{u}})^2, \quad 1 \le t \le n,$$

with $w_{ti} = \left(\sum_{j=1}^{n} K_{tj}\right)^{-1} K_{ti}$, $K_{ti} = K((t-i)/nb)$ if $t \neq i$ and $K_{ii} = 0$, where $K(\cdot)$ is a kernel function on the real line and b is the bandwidth. The following assumption is needed for our variance estimator.

Assumption A2. (i) The kernel $K(\cdot)$ is a bounded density function defined on the real line such that $K(\cdot)$ is nondecreasing on $(-\infty, 0]$ and decreasing on $[0, \infty)$ and $\int_{\mathbb{R}} v^2 K(v)dv < \infty$. The function $K(\cdot)$ is differentiable except a finite number of points and the derivative $K'(\cdot)$ satisfies $\int_{\mathbb{R}} |xK'(x)|dx < \infty$. Moreover, the Fourier Transform $\mathcal{F}[K](\cdot)$ of $K(\cdot)$ satisfies $\int_{\mathbb{R}} |s|^{\tau}\,|\mathcal{F}[K](s)|\,ds < \infty$ for some $\tau > 0$.

(ii) The bandwidth b is taken in the range $\mathfrak{B}_n = [c_{\min} b_n, c_{\max} b_n]$ with $0 < c_{\min} < c_{\max} < \infty$ and $nb_n^{4-\gamma} + 1/nb_n^{2+\gamma} \to 0$ as $n \to \infty$, for some small $\gamma > 0$.

Let $\hat{\epsilon}_t = (\hat{u}_t - \overline{\hat{u}})/\hat{h}_t$. We are now ready to consider the following JB test statistic:

$$Q_{JB}^{\epsilon} = n\left[Q_{JB}^{S,\epsilon} + Q_{JB}^{K,\epsilon}\right],$$

where

$$Q_{JB}^{S,\epsilon} = \frac{\hat{v}_3^2}{6\hat{v}_2^3} \quad \text{and} \quad Q_{JB}^{K,\epsilon} = \frac{1}{24}\left(\frac{\hat{v}_4}{\hat{v}_2^2} - 3\right)^2,$$

with $\hat{v}_j = n^{-1}\sum_{t=1}^{n}\hat{\epsilon}_t^j$. The following proposition gives the asymptotic distribution of Q_{JB}^{ϵ}.

Proposition 1. *Under the assumptions A0, A1 and A2, we have as $n \to \infty$*

$$Q_{JB}^{\epsilon} \Rightarrow \chi_2^2, \tag{2.7}$$

uniformly with respect to $b \in \mathfrak{B}_n$.

Proposition 1 can be proved using the same arguments given in Yu (2007), together with those of the proof of Proposition 4 in Patilea and Raïssi (2014). Therefore we skip the proof. For building a test using the above result, we suggest to choose the bandwidth by minimizing the cross-validation (CV) criterion (see Wasserman (2006, p69-70)). On the other hand several kernels available in the literature can be used. In the numerical experiments section below, we consider the Gaussian kernel and choose the bandwidth by CV unless otherwise specified. The test obtained using (2.7) and choosing the bandwidth by cross-validation is denoted by T_{cv}. The standard test that does not take into account the unconditional heteroscedasticity is denoted by T_{st}.

For high frequency time series it is reasonable to suppose that the approximation (2.7) is satisfactory when the bandwidth is carefully chosen. Nevertheless considering the above sophisticated procedure for small n is questionable. Therefore we propose to apply the following parametric bootstrap algorithm inspired from Francq and Zakoïan (2010, p335).

1 Generate $\epsilon_t^{(b)} \sim \mathcal{N}(0, 1)$, $1 + p \le t \le n$, build the bootstrap errors $u_t^{(b)} = \epsilon_t^{(b)}\hat{h}_t$, and the bootstrap series $y_t^{(b)}$ using (2.1), but with $\hat{\omega}$ and $\hat{\theta}(\hat{\omega})$ (see (2.2) and (2.3)).