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Fitting and forecasting yield curves with a mixed-frequency affine model: Evidence from China

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ABSTRACT

This paper proposes a novel mixed-frequency affine term structure model to improving the fit and forecasting ability of yield curves. We also show the Bayesian estimation method related to this mixed-frequency model. Then we conduct an empirical study using Chinese macro and financial data. The empirical results show that compared with the traditional same-frequency affine model, the mixed-frequency affine model offers superior performance for fitting the yield curve and term structure factors. Specifically, this mixed-frequency affine model can provide more accurate out-of-sample forecast results of the yield curve.

1. Introduction

The yield curve plays important role in macroeconomic behavior because it often contains useful information about real economic activity and inflation (Levant and Ma, 2016). For example, the slope of the yield curve is usually considered a crucial indicator to forecast future economic conditions. Numerous theoretic and empirical researches have focused on the specification of term structure model to investigate the yield curve behavior (Duffie and Kan, 1996; Diebold et al., 2006; Christensen et al., 2011; Kaya, 2013). Many of them suggest that correctly constructing the model is crucial to improving the fit and forecasting ability of yield curves.

A wide variety of term structure models have been proposed in the literature. One of the most popular models is an affine term structure (ATS) model (Duffie and Kan, 1996; Dai and Singleton, 2002). Dai and Singleton (2002) analyze ATS models and show that the yield curve's movements can be reduced to three factors. In addition to the affine model, another type of term structure model is the dynamic Nelson–Siegel (NS) model (Nelson and Siegel, 1987; Diebold et al., 2006). More recently, Christensen et al. (2011) place the NS model in a theoretically consistent arbitrage-free framework. Kaya (2013) uses the NS model to forecast the yield curve in Turkey. Paccagnini (2016) employs the NS model to study macroeconomic determinants of the US term structure during the Great Moderation.

Considering a close relation between the yield curve and macroeconomic variables, a number of researchers have advocated building an affine macro-finance term structure model in recent years (Ang and Piazzesi, 2003; Rudebusch and Wu, 2008; Spreij et al., 2011; Favero et al., 2012; Joslin et al., 2014). Ang and Piazzesi (2003) investigate possible empirical linkages between macroeconomic variables and bond prices using this model. Favero et al. (2012) investigate the forecasting performance of the NS and affine macro-finance term structure model with macroeconomic variables and find that macro factors are very useful in forecasting medium and long rates. Joslin et al. (2014) develop a novel affine macro-finance term structure model and show that output and inflation risks account for a large portion of the variation in forward interest rate risk premiums.

The aforementioned affine macro-finance model provides a useful framework to fit (forecast) the yield curve and better understand its interactions with macroeconomics. However, to the best of our knowledge, many research models have been limited to the same frequency. This limitation means that yield curve modeling cannot utilize all available information since the macro and financial data are usually observed with different (mixed) frequencies. Many macro indicators are released with monthly and quarterly frequency (e.g., quarterly GDP), but financial observations are published with daily or even higher frequency. As a result, some crucial variable with a different frequency can fail to be introduced into the model. Unfortunately, the loss of important mixed-frequency information may be crucial when estimating or forecasting indicators (Fuleky and Bonham, 2013). Therefore, how to specify a term structure model that incorporates the different frequency variables remains an unsolved problem.

This paper develops a novel mixed-frequency macro-finance affine term structure model to fill this gap. The main aims of this paper are as follows. First, we specify the mixed-frequency macro-finance affine model with a Nelson–Siegel representation and rewrite this model as

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state-space model for easy estimation. Second, the Bayesian estimation method related to this mixed-frequency model is studied. We show detailed steps of MCMC sampling. Third, compared with same-frequency models, we discuss the performance of fitting and forecast of mixed-frequency models using the Chinese macro and financial data. We also test the power of our mixed-frequency model forecasts by using Diebold and Mariano statistics.

Our contributions to the literature are as follows. First, we propose a novel mixed-frequency macro-finance affine term structure model. This work extends and enriches the specification of term structure models. Second, this mixed-frequency model shows better in-sample fitting performance than that of same-frequency models via empirical research. Specifically, this mixed-frequency model can provide more accurate out-of-sample forecast results. This finding helps us to understand the role of macroeconomics to improving the fit and forecasting ability of yield curves.

The remainder of this paper is organized as follows. We introduce the traditional same-frequency model and present the specification and estimation method of mixed-frequency term structure model in Section 2. In Section 3, we describe low-frequency macro variables and high-frequency government bonds in China. Section 4 provides and analyzes the parameter estimation and other empirical results. In Section 5, we present concluding remarks.

2. The model

2.1. Dynamic Nelson–Siegel model

There are many term structure models in the literature. A popular representation of the cross-section of yields at any point in time is given by the Nelson and Siegel (1987) curve:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{1}$$

where β_1, β_2, β_3, and λ are parameters, τ denotes maturity, and $y(\tau)$ is the set of yields. Moreover, Diebold and Li (2006) show that the Nelson–Siegel representation can be interpreted as a latent factor model in which β_1, β_2 and β_3 are the time-varying level, slope, and curvature factor, respectively. We write the multiple maturities of the Nelson–Siegel curve in the following form:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} L_t-\mu_L \\ S_t-\mu_S \\ C_t-\mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1}-\mu_L \\ S_{t-1}-\mu_S \\ C_{t-1}-\mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix} \tag{3}$$

The model can be further written into the state space model.

$$y_t = \Lambda f_t + \varepsilon_t \tag{4}$$

$$f_t - \mu_1 = A_1(f_{t-1} - \mu_1) + \eta_t \tag{5}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix}\right] \tag{6}$$

where $y_t = [y_t(\tau_1), \cdots, y_t(\tau_N)]'$ is the $N \times 1$ vector of the bond yield. $f_t = [L_t \;\; S_t \;\; C_t]'$ is the 3×1 term structure factor, and L_t, S_t and C_t capture the level, slope and curvature factor of the term structure, respectively. The error terms ε_t and η_t are assumed to be independently and identically distributed white noise. Λ is the $N \times 3$ coefficient matrix of the measurement equation, and μ_1 and A_1 are the coefficient matrices of the state equation.

Diebold et al. (2006) specify the dynamic factor Nelson–Siegel model with macroeconomic indicators. Let $x_t^{(m)}$ be monthly macroeconomic indicators (e.g., CPI, IP), and t is denoted as the monthly frequency. Diebold et al. (2006) assume that the macroeconomic indicators do not directly affect the yield curve $y_t^{(m)}$, so that the model can be written as:

$$y_t^{(m)} = \Lambda f_t^{(m)} + \varepsilon_t^{(m)} \tag{7}$$

$$\begin{pmatrix} f_t^{(m)} - \mu_f \\ x_t^{(m)} - \mu_x \end{pmatrix} = A_2 \begin{pmatrix} f_{t-1}^{(m)} - \mu_f \\ x_{t-1}^{(m)} - \mu_x \end{pmatrix} + \begin{pmatrix} \eta_{f,t}^{(m)} \\ \eta_{x,t}^{(m)} \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} \varepsilon_t^{(m)} \\ \eta_t^{(m)} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H & 0 \\ 0 & \Omega \end{pmatrix}\right] \tag{9}$$

The state Eq. (8) governs the dynamic relationship between the term structure factors and macroeconomic indicators.

2.2. Affine Nelson–Siegel model

The specification of affine term structure models is more complicated. However, affine models satisfy the hypothesis of risk-free arbitrage that is the fundamental principle in the financial literature. Here, we mainly focus on the affine model with Nelson–Siegel representation.

Christensen et al. (2011) have developed a continuous-time affine Nelson–Siegel model that satisfies the following assumptions:

The instantaneous risk-free rate is assumed to be the affine function of state variables:

$$r_t = \delta_0 + \delta_1{}' X_t \tag{10}$$

where X_t is the vector of state variables.

Under the physical measure (*P-measure*), the state variables that consist of three latent variables can be written as the following process:

$$dX_t = K^P(\Theta^P - X_t)dt + \Sigma dW_t^P \tag{11}$$

where K^P is the mean-reversion matrix, Θ^P represents any mean vector, and W_t^P is the standard Brownian motion.

Under the risk-neutral measure (*Q-measure*), the state variables are described by the following equation:

$$dX_t = K^Q(\Theta^Q - X_t)dt + \Sigma dW_t^Q \tag{12}$$

where K^Q satisfies

$$K^Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{bmatrix} \tag{13}$$

Risk price Γ_t has the following affine specification under the *P-measure*:

$$\Gamma_t = \gamma_0 + \gamma_1 X_t \tag{14}$$

The relationship between the real-world dynamics under the *P-measure* and risk-neutral dynamics under the *Q-measure* is given by the measure change.

$$dW_t^Q = dW_t^P + \Gamma_t dt \tag{15}$$

The specification of the continuous-time affine Nelson–Siegel model proposed by Christensen et al. (2011) can be written as:

$$y_t(\tau_n) = -\frac{1}{\tau_n}(A_n + B'_n X_t) \tag{16}$$

$$X_t = (I - exp(-K^P\Delta t))\theta^P + exp(-K^P\Delta t)X_{t-1} \tag{17}$$

where