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A dynamic Nelson-Siegel yield curve model with Markov switching☆

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ABSTRACT

This paper proposes a model to better capture persistent regime changes in the interest rates of the US term structure. While the previous literature on this matter proposes that regime changes in the term structure are due to persistent changes in the conditional mean and volatility of interest rates we find that changes in a single parameter that determines the factor loadings of the model better captures regime changes. We show that this model gives superior in-sample forecasting performance as compared to a baseline model and a volatility-switching model. In general, we find compelling evidence that the extracted factors from our term structure models are closely related with various economic variables. Furthermore, we investigate and find evidence that the effects of macroeconomic phenomena such as monetary policy, inflation expectations, and real economic activity differ according to the particular regime realized for the term structure. In particular, we identify the periods where monetary policy appears to have a greater effect on the yield curve, and the periods where inflation expectations seem to have a greater effect in yield determination. We also find convincing evidence of a relationship between the regimes estimated by the various switching models with economic activity and monetary policy.

1. Introduction

The yield curve often contains useful information about real economic activity and inflation. For example, the level factor (the long-term yield-to-maturity) is often argued to be closely related with inflation expectations, while the steepness or the slope factor (the long-term yield-to-maturity minus the short-term yield-to-maturity) has been shown to vary with the business cycles and is heavily influenced by monetary policy (see Evans and Marshall (1998), and Wu (2002)). The most recent monetary policies, such as Operation Twist conducted by the Federal Reserve Bank in an attempt to lower the long-term interest rate and raise the short-term rate, directly work on the yields curve and serve as a great example of how the yield curve, instead of just one single policy rate–federal funds rate–is expected to have a significant impact on the economy. As such, it is important to correctly model the yield curve to understand better its interactions with business cycles, and the monetary policy transmission mechanism through its impact on the yield curve.

The interaction of the term structure and the macroeconomy has been investigated by a growing work of empirical literature. Examples include (but are not limited to) Diebold and Li (2006), hereafter DL) and Diebold et al. (2006), hereafter DRA) who employ a generalized version of the Nelson and Siegel (1987), hereafter NS) yield curve model. More recently, Christensen et al. (2011) place the NS model in a theoretically consistent arbitrage-free framework. In this paper, we rely on the results of Coroneo et al. (2011) who finds the NS model is close to being arbitrage-free when applied to the US market, although it does not explicitly impose these restrictions.

Another stream of literature has shown the US interest rate dynamics of the term structure to be subject to frequent regime changes (see Bansal and Zhou (2002)). Although some regime changes are results of obvious changes in monetary policy as in the Volcker era and obvious changes in business cycle conditions such as the oil supply shock of the 1970s, there are many other regime changes that are due to more frequent business cycle fluctuations and often indirectly observed changes in the financial markets. Chauvet (1998) introduces regime switching to a dynamic factor model of business cycle fluctuations and thus accurately captures asymmetries associated with economic expansions and contractions. Startz and Tsang (2010) incorporate Markov regime switching into an trend/cycle unobserved components model of the yield curve to account for regime changes of the yield curve. Abdymomunov and Kang (2015) find the differences in

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the term spread across regimes is explained through the term premia rather than expectations of future short rates. Capturing regime changes in order to model the dynamic movements of the yield curve more accurately is becoming a growing source of investigation in the term structure literature as seen in Xiang and Zhu (2013) and Hevia et al. (2015). In particular, our paper differs from Hevia et al. (2015) in a number of important dimensions. First, Hevia et al. (2015) did not report the estimation results of the model with Markov-switching volatility while we find such a model provides important insights for modeling the yields curve. We also conducted statistical exercises to account for the Davies' nusance parameter issue in efforts to formally test for the significance of the Markov-switching model relative to the baseline no-switching model (Section 4.5), which Hevia et al. did not attempt to do. Finally, we document the important connections between the regime switchings and macroeconomic indicators using logit models in Section 4.6. In sum, our work made a number of important contributions that are outside the scope of Hevia et al. (2015).

In this paper, we model the parameter instability in the term structure and relate the regime switching to economic fundamentals by applying a Markov-switching component to the factor loading parameter which controls the influence of the slope and curvature yield factors on yields. As mentioned previously, the literature has related the slope factor to monetary policy. Also, it has been shown that the curvature factor is heavily influenced by monetary policy as well (see Dewachter and Lyrio (2008) and Bekaert et al. (2010)). In the extant literature, concerning the factor loading parameter, this parameter has been primarily utilized in improving the forecasting ability of the NS model (see Svensson (1995)), Christensen et al. (2009), Koopman et al. (2010)). By assuming the factor loading parameter follows a two-state Markov-process we are able to improve the forecasting ability of the NS model while gaining insight into regime changes of the term structure through the macroeconomic fundamental variables, inflation expectations and monetary policy. Recently, Yu and Salyards (2009) and Yu and Zivot (2011) apply a dynamic NS model to modeling corporate bond yields and they find that the optimal factor loading parameter, changes as one goes from modeling investment to speculative grade bonds. Their results corroborate our findings in general.

We contribute to the literature by introducing and thoroughly evaluating regime-switching factor loadings and regime-switching volatility in the dynamic Nelson-Siegel model. In our models, regimes are characterized by a latent Markov switching component—the fourth latent factor. We apply a Markov switching component to the loading parameters of the factors as well as the factors' volatility. Comparisons between the models are made by presenting goodness-of-fit statistics and AIC/BIC values. We also implement the Likelihood Ratio (LR) tests to investigate if our models are statistically different from the baseline linear DL model. Although both models are found to be statistically different from the baseline model, the root mean square error (RMSE) analysis shows the model with the loading parameter switching yields the smallest RMSE across the short, medium, and long maturity ranges and in terms of overall fit. This model also gives the minimum AIC/BIC values of all models under consideration.

In light of recent discussions about potential interactions between the interest rates factors and the macro-economy, we investigate the relationship between the extracted factors from our DNS models and the observed macroeconomic variables. We find that our interest rate factors, which are extracted separately from the macroeconomic variables, are closely related with the macro-economy. Specifically, we find the level factor is strongly correlated with the inflation expectation, and the slope factor appears to be counter-cyclical, which is consistent with the finding by Wu (2002) that the slope factor is related with monetary policy. Furthermore, in the regime-switching DL model we find that the loading of the slope factor on the yield curve is larger during recessions than expansions. This seems to suggest an asymmetric effect of the monetary policy on the yield curve over business cycles.

This paper is organized as follows. Section 2 describes the baseline dynamic Nelson-Siegel model and the regime-switching DNS model. Section 3 describes the data. Section 4 presents and discusses the estimation results. Section 5 concludes. Appendices discuss the estimation procedure via Kalman filter (KF) and the Kim algorithm (KA).

2. Models and estimation

In this section we introduce the baseline dynamic Nelson-Siegel (DNS) model. The appeal of the DNS model lies in its extension to the time dimension. We also introduce our regime-switching extensions of the DNS models and the estimation technique used.

2.1. The Dynamic Nelson-Siegel Model

The Diebold and Li (2006) factorization of the NS model is given by

$$y_t(m)=y_t(m;\boldsymbol{F}_t,\lambda)=L_t+S_t\frac{1-e^{-\lambda m}}{\lambda m}+C_t\left(\frac{1-e^{-\lambda m}}{\lambda m}-e^{-\lambda m}\right) \tag{1}$$

where $\boldsymbol{F}_t=(L_t, S_t, C_t)'$ is a vector representing level, slope, and curvature of the yield curve, for given time t, maturity m, and constant λ, the factor loading parameter. This is the baseline DNS model in our analysis.

The shape of the yield curve comes from the factor loadings and their respective weights in F_t. From Eq. (1), the factor loading associated with L_t is assumed to be unity for all maturities and therefore influences short, medium, and long-term interest rates equally. The loading factors for S_t and C_t depend on both maturity and the loading parameter. For a given t, the slope factor loading converges to one as $\lambda \downarrow 0$ (or $m \downarrow 0$) and converges to zero as $\lambda \to \infty$ (or $m \to \infty$). The curvature factor loading converges to zero as $\lambda \downarrow 0$ (or $m \downarrow 0$) and as $\lambda \to \infty$ (or $m \to \infty$) for a given t.

Since we are interested in the loading parameter's effect on yields, we use the limit analysis above to understand the asymptotic behavior of the yield curve. The yield curve converges to $L + S$ as $\lambda \downarrow 0$ and converges to L as $\lambda \to \infty$ for a given t. These limiting values indicate that without the loading parameter the yield curve is flat and with extreme values for the loading parameter the yield curve would become flat. So both "reasonable" values for λ *and* the level factor are responsible for the wide range of non-flat yield curve shapes within an NS framework.

2.2. DNS model estimation

We adopt the DRA state-space framework to model each variant of the NS model in this paper. Our measurement equation models the time-series process of the yields according to the latent factors and takes the form

$$\begin{pmatrix} y_t(m_1)\\ y_t(m_2)\\ \vdots \\ y_t(m_N)\end{pmatrix}=\begin{pmatrix} 1 & \frac{1-e^{-\lambda m_1}}{\lambda m_1} & \frac{1-e^{-\lambda m_1}}{\lambda m_1}-e^{-\lambda m_1}\\ 1 & \frac{1-e^{-\lambda m_2}}{\lambda m_2} & \frac{1-e^{-\lambda m_2}}{\lambda m_2}-e^{-\lambda m_2}\\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N}-e^{-\lambda m_N}\end{pmatrix}\begin{pmatrix} L_t\\ S_t\\ C_t\end{pmatrix}+\begin{pmatrix} \varepsilon_t(m_1)\\ \varepsilon_t(m_2)\\ \vdots \\ \varepsilon_t(m_N)\end{pmatrix} \tag{2}$$

or expressed in matrix notation as

$$\boldsymbol{y}_t=\boldsymbol{\Lambda}(\lambda)\boldsymbol{F}_t+\boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t\sim MN(0,\boldsymbol{\Sigma}_\varepsilon), t=1,\dots, T \tag{3}$$

with $\boldsymbol{y}_t$ representing the $N \times 1$ vector of yields, $N \times 3$ factor loading matrix $\boldsymbol{\Lambda}(\lambda)$, 3×1 latent factor vector $\boldsymbol{F}_t$, and $N \times 1$ yield disturbance vector $\boldsymbol{\varepsilon}_t$ (or so-called measurement errors of the yields). The diagonal structure of $\boldsymbol{\Sigma}_\varepsilon$ implies that measurement errors across maturities of $\boldsymbol{y}_t$ are uncorrelated and is a fairly standard assumption in the literature. The transition equation, which models the time series process of the