ARTICLE IN PRESS

Economic Modelling (xxxx) xxxx–xxxx

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Reserve modelling and the aggregation of risks using time varying copula models

Sawssen Araichi[a,b,*], Christian de Peretti[a], Lotfi Belkacem[b]

[a] Laboratory of Actuarial and Financial Sciences (LSAF, EA2429) Institute of Financial and Insurance Sciences, University Claude Bernard Lyon 1, France
[b] Laboratory Research for Economy, Management and Quantitative Finance, Institute of High Commercial Studies of Sousse, Tunisia

ARTICLE INFO

JEL classification:
C22
C52
C58
G22

Keywords:
Claims reserving
Time varying copula models
Generalized autoregressive conditional sinistrality model
Simulation method
Solvency capital requirement

ABSTRACT

This paper is concerned with the appropriate claim reserving modelling and aggregation of risks in the insurance sector. In fact, literature review provided some methods to evaluate the total amount of reserves and solvency capital of different lines of business. However, these models were derived under the independent losses assumption. Thus, the total amount of reserves and capital may be inaccurate when losses are dependent, as it is the case in practice. In this paper, a novel model is proposed aiming to handle temporal dependence, both between a line of business claim's amounts and between the two lines of business claims. Generalized Autoregressive Conditional Sinistrality model is used to analyze the evolution in time of dependence and time varying copula functions are proposed to aggregate risks. To achieve such purpose, a simulation study, highlighting the impact on reserves and Solvency Capital Requirement, is performed. Results revealed that a diversification effect could be gained on the Solvency Capital when considering time varying dependence structures.

1. Introduction

It is an acknowledged fact that forecasting claims reserves and assessing the accurate solvency capital are some of the most crucial issues for actuaries.

Indeed, insurance companies must evaluate a total amount of capital and reserves, that allows them to participate in the capital market development and to cover their engagements towards the policyholders. Also, serious companies should have a quick response time, be adept at handling claims and have excellent customer service.

In the context of non life insurance, it is a general presumption in classical approaches that claims of different lines of business are independent. However, it is unlikely that claims of different policies are independent. To further illustrate, a car accident can generate claims on both auto and health policies. Also, a fire may spread from one building to another, resulting in claims from two or more policyholders. Thus, one event may involve a series of claims. In such context, assuming that different policies of lines of business are independent can lead to an over or under estimation of the aggregate loss. It has been pointed out that, methods enabling modelling dependencies of different lines of business policies are needed, in order to improve the estimation accuracy of the total reserves and solvency capital.

Copulas functions have been proposed in several actuarial studies as a relevant tool when handling different dependence structures. In fact, they have been introduced to the actuarial mathematics by Frees and Valdez (1998). Since then, a large number of investigations have applied copulas functions to fully capture a wide range of dependence structures amongst different insured risks. For instance, Frees and Wang (2006) have used copulas to model dependence between claims, in pursuance of aggregate losses' credibility predictors. In addition, Kaishev and Dimitrova (2006) have disclosed the importance of copulas in reinsurance by modelling the claim severities using the copula function. Antonin and Benjamin (2001) also used copulas to assess the reserve amount, by providing a model that combines simultaneously the copula theory and credibility theory, aiming the detection of the dependence between the lines of business. Furthermore, Belguise and Levi (2002); Faivre (2002) and Cadoux and Loizeau (2004) have shown that copulas models allow to capture dependence between lines of business and yield a higher amount of capital than when assuming independence. Besides, Bargés et al. (2009) evaluated the capital allocation for the overall portfolio, using the TVaR as a measure of risk and the Farlie–Gumbel–Morgenstern (FGM) copula. What is more, Diers et al. (2012) have attested to the flexibility of a Bernstein copula when modelling several lines of business. Also, Zhang and Dukic (2013) have introduced a Bayesian

* Corresponding author at: Laboratory Research for Economy, Management and Quantitative Finance, Institute of High Commercial Studies of Sousse, Tunisia
E-mail address: sawssen.araichi@gmail.com (S. Araichi).

http://dx.doi.org/10.1016/j.econmod.2016.11.016
Received 31 May 2016; Received in revised form 18 October 2016; Accepted 21 November 2016
0264-9993/ © 2016 Published by Elsevier B.V.

multivariate model, based on the use of parametric copula to model dependencies between various lines of insurance claims.

In more recent actuarial researches, a more relevant technique, namely the copula regression model, where marginal distributions are specified conditional on a set of covariates, was introduced. Some recent applications include the modelling of insurance claims (e.g Frees and Valdez, 2008; Frees et al., 2009). The prediction of reserves (e.g Zhao and Zhou, 2010; Shi and Frees , 2011). The examination of asymmetric information (e.g Shi and Valdez, 2011, 2012). The analysis of insurance claims (e.g Shi and Frees, 2010; Shi and Valdez , 2012; Krämer et al., 2013), among others.

Despite the popularity of copulas in modelling dependence, their application in time series data in the insurance sector is scarce. The majority of approaches proposed in actuarial science do not involve the temporal dependence problem (i.e. when a serial dependencies and a time varying dependence structure of claims can be captured).

More recently, a model has been proposed by Pešta and Okhrin (2014) pertaining to the time series claims data. Such a model introduced the Generalized time series model for conditional mean and variance of one lines of business, and further elaborates copula approach for dependence modelling within their proposed model. These authors have proved, that their proposed model improves the parameter estimates consistency as well as the reserve distribution precision. Therefore, it is important to consider the temporal dependence structure of claims, by modelling that structure using a model that is able to follow the change in dependence over time. However, their paper only deals with the temporal dependence structure of one line of business, while neglecting the time varying dependence structure of several lines of business.

This paper discusses a new approach to determine the solvency capital requirement of two lines of business. Originally, our research accounts for both, the temporal dependence in each line of business and between the two lines of business. For that, a time series Generalized linear model namely a Generalized Autoregressive Conditional Sinistrality model is presented, to capture temporal dependence within each line of business.

Moreover, aiming to extend the knowledge ensued from prior work, which employed static copulas and presented the dependence structure as a value, this paper used the dynamic copula functions, with time-evolving dependence structure. Therefore, the aggregation of risks is made, taking into account the dynamic claims behavior. To our knowledge and as far as it has been researched, this work presents an early investigation of the application of the dynamic copulas in insurance.

Furthermore, a simulation procedure is provided in order to estimate the aggregate reserves and the total solvency capital.

This paper is organized as follows: Section 2 presents the Autoregressive Conditional Sinistrality Model for marginals modelling. Section 3 provides the time varying copula model. Section 4 reports the empirical results, followed by concluding remarks and some open questions in Section 5.

2. Marginal models

2.1. Generalized autoregressive conditional sinistrality model

The primary goal of our study is to evaluate reserves and Solvency Capital taking into account the dynamic dependence structure between lines of business. For that, a suitable model for marginals must be identified. In this paper, a Generalized Autoregressive Conditional Sinistrality Model (GACSM) proposed by Araichi et al. (2015) is considered. This model assumes that claim amounts $y_{i,t}$ from the same accident quarter are correlated. A dynamic specification is considered

$$y_{i,t} = \psi_{i,t}\epsilon_{i,t}, \tag{1}$$

$$\epsilon_{i,t} \sim \text{i. i. d. negative variable with } E(\epsilon_{i,t}) = 1. \tag{2}$$

where $\epsilon_{i,t}$ denote the standardized claim amounts (residuals), that are assumed to belong to the exponential family.

The conditional expectation of $y_{i,t}$ is denoted as: $E(y_{i,t}|\Omega_{i,t-1}) = \psi_{i,t} = [\psi_{i,1}, \ldots, \psi_{i,T_i}]$, where $\Omega_{i,t-1} = \{x_{i,1}, x_{i,2}, \ldots, x_{i,t-1}, y_{i,1}, y_{i,2}, \ldots, y_{i,t-1}\}$ is the past information until the development quarter $t-1$.

This conditional expectation of claim amounts is related to the predictor $\eta_{i,t}$ by the link function such that $g(\psi_{i,t}) = \eta_{i,t}$. As pointed out by Merz and Wüthrich (2008) a log link is typically a natural choice in the insurance reserving context.

Here, the problem of temporal dependence of development quarters is considered. For that, we suppose a time dependent specification for $\eta_{i,t}$ defined by

$$\eta_{i,t} = X'_{i,t}\beta + \sum_{j=1}^{p}\theta_j\{g(y_{i,t-j}) - X'_{i,t-j}\beta\} + \sum_{j=1}^{q}\phi_j\{g(y_{i,t-j}) - \eta_{i,t-j}\} \tag{3}$$

where $X'_{i,t}$ is a $z \times d$ matrix of dummy covariates that arranges the impact of accident and development quarter on the claim amounts through model parameter $\beta \in \mathbb{R}^{d\times 1} = [\gamma, \alpha_2, \ldots, \alpha_n, \delta_2, \ldots, \delta_{T^i}]$, where α_i stands for the effect of accident quarter i, and δ_t represents the effect of the development quarter t, and γ is the constant parameter of the model (taking $\alpha_1 = \delta_1 = 0$ to avoid over parametrization).

Standardized claim amounts $\epsilon_{i,t}$ are assumed to follow a Gamma distribution with $E(\epsilon_{i,t}) = 1$. Indeed, in Araichi et al. (2016) it was already proven by the authors that the Gamma model is adequate for the line Auto Damage and Auto Liability.

Given $\Omega_{i,t-1}$, the distribution and the density function of the Gamma GACSM are presented respectively as

$$\forall\ i = 1\ldots.n,\ \ t = 2,\ \ldots.,T_i,\ \ T_i = T + 1 - iF(y_{i,t}) = \frac{\Gamma\left(k, \dfrac{y_{i,t}}{\psi_{i,t}}\right)}{\Gamma(k)} \tag{4}$$

$$f\left(y_{i,t}\middle|\Omega_{i,t-1}\right) = \frac{y_{i,t}^{k-1}k^k}{\psi_{i,t}^k\Gamma(k)}\exp\left(-\frac{y_{i,t}^k}{\psi_{i,t}}\right) \tag{5}$$

where k is the shape parameter, $\psi_{i,t} = exp(\eta_{i,t})$ and $\eta_{i,t}$ is given by

$$\eta_{i,t} = X'_{i,t}\beta + \sum_{j=1}^{p}\phi_j\{\log(y_{i,t-j}) - X'_{i,t-j}\beta\} + \sum_{j=1}^{q}\theta_j\{\log(y_{i,t-j}) - \eta_{i,t-j}\} \tag{6}$$

The log likelihood of the Gamma GACSM is then formalized as

$$L\left(y_{i,t}\middle|\Omega_{i,t-1}\right) = \sum_{t=1}^{T_i}\left((k-1)\log(y_{i,t}) + k\log(k) - k\log(\psi_{i,t}) - \log(\Gamma(k)) - \frac{y_{i,t}k}{\psi_{i,t}}\right) \tag{7}$$

The parameters of GACSM are estimated by maximizing the log likelihood function, the procedure yields a consistent and asymptotically normal estimates (see Section 2.2).

2.2. Convergence of the maximum likelihood estimator

Let $\theta = (\beta', \theta_1, \ldots, \theta_p, \phi_1, \ldots, \phi_q)' \in \Theta$ denote the vector of all the parameters of the model and belongs to a parameter space Θ.

Assumption 1. θ_0, the true value of θ, belongs inside Θ (i.e. excluding the frontier of Θ).

Assumption 2. f_{θ_ϵ} is continuously differentiable with respect to θ_ϵ.

Property 1. *Assumptions 1 and 2 imply that the log-likelihood is continuously differentiable with respect to θ.*