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Economic growth cycles driven by investment delay

Adam Krawiec[a,c,*], Marek Szydłowski[b,c]

[a] Institute of Economics, Finance and Management, Jagiellonian University, Łojasiewicza 4, 30-348 Kraków, Poland
[b] Astronomical Observatory, Jagiellonian University, Orla 171, 30-244 Kraków, Poland
[c] Mark Kac Complex Systems Research Centre, Jagiellonian University, Łojasiewicza 11, 30-348 Kraków, Poland

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A B S T R A C T

We study the model of growth cycles in the framework of the Keynesian macroeconomic theory. The Kaldor–Kalecki growth model is the Kaldor business cycle model with two modifications: exponential growth introduced by Dana and Malgrange and Kaleckian investment time delay considered in this paper. This model has a form of a system of differential equations with time delay. The methods of dynamical system theory and bifurcation theory are used in the analysis of dynamics of growth cycles. Taking the time delay in investment as a bifurcation parameter we show the existence of a single Hopf bifurcation. We show that the time delay creates a limit cycle in a wider range of parameters than the model without the time delay. The time delay is the source of fluctuations which are described in a deterministic way. We carry out the numerical analysis of the model and find the limit cycle solution as well as we find that the higher the time delay parameter value the longer the period and the higher the amplitude of income and capital. We find that there is a distinguished value of growth rate parameter g=0.0147 for which the lowest value of investment time delay or the lowest value of speed of adjustment is required to obtain cyclic solutions.

1. Introduction

There is much interest in the cyclic phenomena in economics. While many propositions have been formulated to identify the sources of economic fluctuations, we are interested in endogenous, not exogenous, mechanisms. In this paper we study one specific mechanism, investment lags, which is responsible for economic oscillations. In an economic model they are an inherent property which generates endogenous persistent cycles.

The investment lags, also referred to as investment time delay or the time-to-build, can be one of the simplest causes of macroeconomic fluctuations. One of the earliest models of business cycles which took into account the time of investment lags was the Kalecki (1935) model. Kalecki argued that the investment process is made of investment decisions, production of investment goods and deliveries of finished industrial equipment, and assumed that the average time of capital production is T. His explanation of the business cycle mechanism was such that the change of the volume of investment orders precedes the change of the volume of capital stock and the deliveries of new equipment are always above or below the demand for restoration of the industrial equipment. That is why the existing industrial equipment (capital stock) changes cyclically in a persistent way (Kalecki, 1935, pp. 341–342).

The several empirical studies gave us the estimation of the length of the time-to-build period. Mayer (1960) found that the completion of construction project takes on average 15 months. Montgomery (1995) used data for private nonresidential construction projects in the USA and obtained average time of 16.7 months in period 1961–1991.

Another mechanism of cyclic fluctuations was proposed by Kaldor (1940). In his model of business cycles Kaldor paid attention to the varying propensity to invest with respect to income, i.e., he assumed the s-shaped nonlinear investment function. Then the Kaldor–Kalecki model was derived from these two approaches to business cycles modelling (Krawiec and Szydlowski, 1999). This unified model is implemented by means of second-order differential delay equation or equivalently an infinite-dimensional dynamical system. The main feature of this model is a distinction between the investment decisions and the delivery of final goods. The time between starting and finishing investment is described by the time delay parameter. Time delay is responsible for the Hopf bifurcation to a limit cycle solution and for generating cycles, which is a useful way of business cycles modelling. This is a main difference between the Kaldor model and the Kaldor–Kalecki model. In the former cycles are generated only due to the nonlinear investment function. The investment lag varies from months to several years but an averaged lag is used in the Kaldor–Kalecki model (Laramie et al., 2004). This model was investigated extensively

* Corresponding author at: Institute of Economics, Finance and Management, Jagiellonian University, Łojasiewicza 4, 30-348 Kraków, Poland.
E-mail addresses: adam.krawiec@uj.edu.pl (A. Krawiec), marek.szydlowski@uj.edu.pl (M. Szydłowski).

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by many authors (Krawiec and Szydlowski, 2001; Szydlowski et al., 2001; Szydlowski and Krawiec, 2005; Takeuchi and Yamamura, 2004; Bischi et al., 2001; Wu and Wang, 2010).

In a similar way, as the Kaldor model (Chang and Smyth, 1971) was extended by Dana and Malgrange (1984) to incorporate the exponential growth and to exhibit growth cycles, we augment our Kaldor–Kalecki model. We incorporate the exponential growth of autonomous demand to model the growth with cyclic behaviour. In the Kaldor growth model, Dana and Malgrange (1984) found that cyclic behaviour was driven by nonlinearity (s-shape) of investment function. In turn, the mechanism of cyclic behaviour in the Kaldor–Kalecki growth model is the same as in the Kaldor–Kalecki business cycle model. It is an endogenous mechanism of time delay in providing the capital goods in an economy which is responsible for a creation of a limit cycle solution. We assume that both investment function I and saving function S are homogeneous functions of degree one in income Y and capital K. We obtain after introducing new rescaled variables the model of growth cycles with a constant time delay parameter related to time-to-build.

We study the problem of growth cycles in the framework of the Keynesian macroeconomic theory. The possibility of modelling the growth through cycles was first proposed by Goodwin (1967). From this seminal paper the growth cycles in different economic context have been extensively studied (Evans et al., 1998; Matsuyama, 1999).

This Kaldor–Kalecki growth model is a system of delay differential equations. This class of equations is suitable for the description of dynamics where past evidence has an impact on the present state of the system (Hale and Verduyn Lunel, 1993). There are some applications of the theory of delay differential equations in microeconomics and macroeconomics (Mackey, 1989; Asea and Zak, 1999). To study the cyclic behaviour we use the bifurcation theory, which offers us some methods of determining the existence of cyclic solutions in models both for ordinary differential equations and delay differential equations (Marsden and McCracken, 1976; Chow and Hale, 1982; Hale and Verduyn Lunel, 1993). In this paper we make use of the Hopf bifurcation theory, which describes a mechanism of creating a limit cycle after a critical point loses its stability when a control parameter changes (Marsden and McCracken, 1976). There are many applications of this bifurcation in different fields of sciences. For example, in economics Benhabib and Nishimura (1979) studied the Hopf bifurcation and the existence of closed orbits in models of optimal economic growth.

In this paper the Kaldor–Kalecki dynamical system is analysed in detail in terms of the infinite dimensional dynamical systems theory and from the point of view of the bifurcation theory. By using these methods it is demonstrated that the presence of cyclic behaviour is caused by the time delay parameter. The scenario of appearing a periodic solution of the system corresponds to the single Hopf bifurcation mechanism. In the phase space the cyclic behaviour is represented by a limit cycle which is characterised by the system parameters, like the speed of adjustment, the time delay parameter, the marginal propensity to save, and the capital depreciation rate.

We carry out the numerical analysis of the Kaldor–Kalecki growth cycle model. We draw the phase portrait to illustrate the limit cycle solutions. We analyse the solutions of the model with respect to the model parameters and find that the higher the time delay parameter value the longer the period and the higher the amplitude of income and capital. We also find that there is a distinguished value of growth rate parameter g=0.0147 for which the lowest value of investment time delay or the lowest value of speed of adjustment is required to obtain cyclic solutions.

The organisation of the text is as follows. In Section 2 we derive the Kaldor–Kalecki growth model equations. In Section 3 we study the existence of cyclic solution through the Hopf bifurcation mechanism. In Section 4 we investigate the asymptotic stability property of the model. The numerical analysis is conducted in Section 5. Conclusion and discussion of obtained results are given in Section 6.

2. The Kaldor–Kalecki growth model

In the framework of the Kaldor (1940) business cycle model we introduced Kalecki's (1935) idea of the delay between the investment decisions and providing the capital goods to the capital accumulation equation and constructed the Kaldor–Kalecki business cycle model (Krawiec and Szydlowski, 1999)

$$\dot{Y}(t) = \alpha[I(Y(t), K(t)) - S(Y(t), K(t))] \tag{1a}$$

$$\dot{K}(t) = I(Y(t-T), K(t)) - \delta K(t). \tag{1b}$$

where the investment function $I(Y, K)$ and the saving function $S(Y, K)$ depend on income Y and capital K, and α is a speed of adjustment in an economy. In this model investment is started at time $t-T$ and finished at time t. Therefore, the time T is the average time of investment completion in the economy. The gross investment $I(Y(t-T), K(t))$ depends on income Y in the past time $t-T$ when investment decisions were made, and on capital K available in the present time t because there is the common knowledge at the decision time $t-T$ about investment which will be finished between $t-T$ and t.

The Kaldor–Kalecki model (1) is the system of differential equations with time delay. While for an ordinary differential equation a solution depends on an initial condition at t_0, an initial function on $[t_0-T, t_0]$ must be given to solve a delay differential equation. It means that to determine future states of a system, its past states for an interval of a delay length T have to be known.

Dana and Malgrange (1984) modified the Kaldor business cycle model by introducing the exponential growth of autonomous demand $G_0 e^{gt}$, where G_0 and g are constant. We imitate their approach in our Kaldor–Kalecki model which leads to the following growth cycle model:

$$\dot{Y}(t) = \alpha[I(Y(t), K(t)) - S(Y(t), K(t)) + G_0 e^{gt}] \tag{2a}$$

$$\dot{K}(t) = I(Y(t-T), K(t)) - \delta K(t). \tag{2b}$$

We call it the Kaldor–Kalecki growth model.

We assume that both $I(Y, K)$ and $S(Y, K)$ are homogeneous functions of degree one with respect to its arguments Y and K. Moreover, without loss of the generality, we choose the saving function to be linear with respect to income Y only

$$S(Y, K) = \gamma Y. \tag{3}$$

On the other hand we preserve the Kaldor assumption about an s-shaped investment function in the form proposed by Dana and Malgrange (1984):

$$I(Y, K) = K\Phi\left(\frac{Y}{K}\right) = K\Phi(x) \tag{4}$$

where a new variable, income per capital $x = Y/K$, is introduced and $\Phi(x)$ is a function of one variable x, which satisfies Kaldor's conditions: $\Phi_{xx}(x) > 0$ for $x < x^*$, $\Phi_{xx}(x^*) = 0$, and $\Phi_{xx}(x) < 0$ for $x > x^*$.

Using the French quarterly data for 1960–1974, Dana and Malgrange obtained the function $\Phi(x)$ in the following form:

$$\Phi(x) = c + \frac{d}{1 + \exp[-a(vx-1)]} \tag{5}$$

or equivalently

$$\Phi(x) = c + \frac{d}{2} e^{\frac{a}{2}(vx-1)} \cosh^{-1}\left[\frac{a}{2}(vx-1)\right]$$

where v=4.23, c=0.01, d=0.026, a=9 (Dana and Malgrange, 1984).

To obtain the stationary solution of System (2) in the form of a constant rate growth path we introduce new variables

$$k = Ke^{-gt}, \quad y = Ye^{-gt}. \tag{6}$$

Now, System (2) in terms of reduced variables (6) takes the form

$$\dot{y}(t) = \alpha[I(y(t), k(t)) - S(y(t), k(t)) + G_0] - gy(t), \tag{7a}$$