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Testing the Gaussian and Student's t copulas in a risk management framework☆

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ABSTRACT

This paper introduces a semiparametric framework for selecting either a Gaussian or a Student's t copula in a d-dimensional setting. We compare the two models using four different approaches: (i) four goodness-of-fit graphical plots, (ii) a bootstrapped correlation matrix generated in each scenario with the empirical correlation matrix used as a benchmark, (iii) Value-at-Risk (VaR) and Expected Shortfall (ES) as risk measures, and (iv) co-Value-at-Risk (CoVaR) and Marginal Expected Shortfall (MES) as co-risk measures. We illustrate this four-step procedure using a portfolio of daily returns of six international stock indices. The VaR results confirm that the t-based copula model is an attractive alternative to the Gaussian. The ES analysis is less conclusive, and indicates that risk managers should jointly use the risk measure as well as the copula model. The results highlight the importance of promoting stress testing rather than ES in the risk management industry, particularly in the aftermath of a financial crisis.

1. Introduction

As copulas allow decoupling the risk associated with a portfolio structure from each individual risk source or factor, copula functions have gained popularity in Value-at-Risk and Expected Shortfall estimations (hereafter VaR and ES respectively). These two risk measures have been analysed extensively and recent studies include, for instance, Gao and Zhou (2016); Berger (2015); Ausín et al. (2014); Chen et al. (2014).

From the numerous criteria used in literature to compare copula models, two copula paradigms can be highlighted. The first focuses on optimizing a quantitative criterion where Bassamboo et al. (2008), for example, aim to decrease the variability of ES estimates. The second copula paradigm aims to detect an interpretable dependence structure among financial data where Malevergne and Sornette (2003), for example, highlight a higher correlation among stocks than currencies by comparing the Gaussian and the Student's t copulas on both types of assets.

Indeed, the boundary between these two paradigms is somewhat blurred in a number of situations. In this paper, our aim is not to remove the ambiguity in the positioning of the two paradigms, but to develop a set of straightforward graphical and inferential tools intended to subject the Gaussian and Student's t elliptical copula models to the test of historical data. More precisely, we aim to answer the following questions embedded in these two paradigms:

1. When each copula model is used as the data generating process over 2,608 data points, which provides the best fit with the historical data?
2. When each copula model is used to calibrate VaR and ES, which provides the best trade-off between bias and variance in relation to both risk measures?

At first glance, extant studies would seem to address such questions, especially in relation to the Gaussian and Student's t copulas. However, in the field of risk management, empirical evidence from studies comparing these two copula models remains either unclear, or even worse, providing strikingly different conclusions. A recent illustrative example is that of Grundke and Polle (2012) who analysed mixed portfolios during the 2008 financial turmoil and were unable to reject both the Gaussian and Student's t hypotheses over this period. Clearly, such a result is in contrast with the well-known stylized fact of

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extreme quantiles dependence in financial markets, particularly during a financial turmoil. The recent study of Chen et al. (2014) compares a number of copula functions commonly used in finance, including the symmetrized Joe-Clayton copula (Patton, 2006), and finds that the copula that best captures the bivariate dependence in a stock-bond returns series is the Student's t copula, both before and after the recent global financial crisis. In a bivariate setting, Berger (2015) backtests the performance of daily VaR forecasts and shows that Student's t copulas lead to solid 95% VaR forecasts, and are therefore considered a sensible choice for applied risk measurement.

Berg (2009) observed that the need for intuitive and informative diagnostic plots for the selection and validation of copula models remained unfulfilled. This led to alternatives to formal GoF tests dedicated to copula assessments, such as those based on the Kolmogorov-Smirnov (KS) and the Cramér-Von Mises (CvM) tests, or the Rosenblatt transformation[1]. Even these tests, however, do not always enable differentiating between different copula assumptions (Grundke and Polle, 2012). Moreover, as Patton (2012) points out, although ”blanket” tests such as KS and CvM are consistent, they provide no guidance on the how the model might be improved.

The main contribution of this article is developing a user-friendly semiparametric framework including both graphical and inferential goodness-of-fit (GoF hereafter) diagnostic tools to compare the Gaussian and Student's t copulas in a d-dimensional setting.

Specifically, the GoF graphical tools we consider here are a useful complement to non-parametric plots, such as the Chi/Kendall plots traditionally used in literature to assess dependence and select suitable copulas (see, e.g., Chen et al., 2014; Boero et al., 2011).

Worth noting is that all tools we propose here can be easily extended to the combination of any copula with marginal distributions. Furthermore, due to their simplicity, our GoF toolkit could provide a highly customizable R-package and complement the R-package copula (version 0.999-10) of Hofert et al. (2014)[2], which may be particularly useful in the risk and asset management fields.

Among the large number of copulas with various properties developed in literature with reference to dependence modelling, we select two elliptical copulas for two reasons.

First, despite the relevant shortcoming of elliptical copulas (imposing a symmetric dependence structure), the Gaussian and Student's t copulas are frequently used as competing copula models in finance literature[3]. Moreover, the Gaussian copula is often taken as the benchmark copula in literature (Kang et al., 2010). Even when a mixture of copulas is used with the aim of gaining flexibility, the normality assumption in both theoretical and empirical studies calls for the inclusion of the normal copula in the mixture and ruling out it out entirely may thus be extreme (Hong et al., 2007).

Second, the Gaussian and Student's t copulas are still widely used in the finance industry. Popular risk-management frameworks including J. P. Morgan's CreditMetrics (Gupton et al., 1997) and Moody's KMV (Bohn and Kealhofer, 2001) incorporate the Gaussian copula. Furthermore, the Gaussian copula features prominently in the successive Basel Accords that regulate capital allocation in financial institutions. The regulatory Basel III Accord (Committee et al., 2010) introduces a new capital charge for Credit Valuation Adjustment (CVA) risk. From this perspective, CVA risk is similar to any market risk and should be managed actively as part of large bank trading books. For convenience and consistency with the Basel models, recent modelling approaches show that dependence of exposure on the counterparty's credit quality can be incorporated in the CVA calculation using a Gaussian copula methodology (Rosen and Saunders, 2012).

[1] For a comprehensive review, see Berg (2009) and Genest et al. (2009).

[2] The R-code for all graphical and inferential GoF diagnostic tools used in this paper is available from the authors on request.

[3] See, for example, de Truchis and Keddad (2016); Berger (2015); Babaei et al. (2015); Koziol et al. (2015); Patton (2012).

The remainder of this paper is structured as follows. Section 2 documents the copula approach in risk management. Section 3 presents the methodology and statistical framework. Section 4 introduces graphical and inferential GoF diagnostic tools applied to the Gaussian and Student's t copula scenarios. We conclude the paper with brief remarks in Section 5.

2. Documenting the copula approach in risk management

There is wide agreement that copula functions are an effective element of risk management. Below we highlight some of the reasons why this tool has become increasingly popular in financial modelling.

Technically speaking, a copula returns the joint probability of events as a function of the marginal probabilities of each event. Given that the copula margins and the dependence structure can be disentangled, the latter can be managed independently from the former. Both from a theoretical and practical standpoint, this makes copulas an attractive tool for modelling multivariate dependencies in different areas of risk management, such as credit or operational risk modelling. As copulas allow decomposing a multivariate distribution into univariate distributions and a non-reducible dependency function, they constitute an appropriate tool to understand risk aggregation (Embrechts et al., 2003). A further interesting property of copulas is that they allow interpolating between extreme cases of dependence, which is why they are also widely used in portfolio stress testing. The application of copulas in multi-objective portfolio optimization problems has gained momentum in recent years. For instance, Babaei et al. (2015) undertake a comprehensive analysis exploiting α-stable distribution together with different specifications and calibrations of parametric copula functions.

For market risk managers, the copula model matters for many different reasons. The most salient is that a portfolio's risk is directly related to the dependence of its components. Numerous empirical studies show that different copulas lead to significantly different assessments of the risk of joint downward movements and diversification benefits, *e.g.*, Kole et al. (2007). The copula model used for the stochastic dependence is thus of primary importance in computing risk measures, such as Value-at-Risk or Expected Shortfall (Grundke and Polle, 2012).

Selecting the right copula model is also important for credit risk managers. As an example, Bassamboo et al. (2008) consider a common shock-based model to measure portfolio credit risk. Contrasting the Student's t copula with the Gaussian copula, they observe that the latter underestimates the probability of large losses compared to the former for a large number of obligors in portfolio. In addition, for small values of correlation, the Gaussian copula model significantly underestimates the loss probability compared to the Student's t copula model.

The appropriate choice of a copula model is even more important when markets are in turmoil. Recently, de Truchis and Keddad (2016) through a copula-HAR analysis provide evidence that dependence is sensitive to market conditions. Their results are informative in terms of risk management as they show that portfolio diversification remains possible for a short-term investment horizon, albeit depending on market conditions as regards time-varying correlations.

The main message from the recent global financial crisis is that the default of a so-called Systematically Important Financial Institution (SIFI) can generate negative externalities on the entire financial system. Given the intrinsic inability of standard risk-measurement tools such as VaR and ES to capture this systemic risk, alternative risk measures have been developed to quantify each firm's contribution to the system's overall risk. This new risk-management challenge is today intensely debated by regulatory institutions and academics. Hakwa et al. (2015) propose a flexible framework to compute such a measure – the CoVaR (Adrian and Brunnermeier, 2011) – in a very general stochastic setting based on copula theory. Against the background of systemic risk analysis, a significant advantage of their formula is that it