Optics and Lasers in Engineering 71 (2015) 9–19

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Noise robustness and parallel computation of the inverse compositional Gauss–Newton algorithm in digital image correlation

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ARTICLE INFO

Article history:
Received 15 November 2014
Received in revised form
18 January 2015
Accepted 5 March 2015
Available online 31 March 2015

Keywords:
Noise robustness
Parallel computation
Inverse compositional Gauss–Newton algorithm
Digital image correlation

ABSTRACT

The inverse compositional Gauss–Newton (IC-GN) algorithm is one of the most popular sub-pixel registration algorithms in digital image correlation (DIC). The IC-GN algorithm, compared with the traditional forward additive Newton–Raphson (FA-NR) algorithm, can achieve the same accuracy in less time. However, there are no clear results regarding the noise robustness of IC-GN algorithm and the computational efficiency is still in need of further improvements. In this paper, a theoretical model of the IC-GN algorithm was derived based on the sum of squared differences correlation criterion and linear interpolation. The model indicates that the IC-GN algorithm has better noise robustness than the FA-NR algorithm, and shows no noise-induced bias if the gray gradient operator is chosen properly. Both numerical simulations and experiments show good agreements with the theoretical predictions. Furthermore, a seed point-based parallel method is proposed to improve the calculation speed. Compared with the recently proposed path-independent method, our model is feasible and practical, and it can maximize the computing speed using an improved initial guess. Moreover, we compared the computational efficiency of our method with that of the reliability-guided method using a four-point bending experiment, and the results show that the computational efficiency is greatly improved. This proposed parallel IC-GN algorithm has good noise robustness and is expected to be a practical option for real-time DIC.

1. Introduction

The technique of digital image correlation (DIC) [1–4] has been widely used for non-contact deformation measurement in the field of experimental mechanics. Improving the performance of DIC has been extensively investigated in different ways, including sub-pixel registration algorithms [5–7], sub-pixel interpolation schemes [8,9], subset size [10], subset shape functions [11,12], camera lens distortion [13,14], and image noise [15,16]. Among these aspects, the selection of the sub-pixel registration algorithm is vitally important. Pan et al. showed that the Newton–Raphson (NR) algorithm would be an optimal approach given its measurement accuracy and stability, if it were not for its slow calculation speed [7]. To speed the calculation of the NR algorithm, an inverse compositional Gauss–Newton (IC-GN) algorithm combined with a zero-mean normalized sum of squared difference (ZNSSD) correlation criterion was recently introduced for DIC [17]. Compared with the traditional forward additive Newton–Raphson (FA-NR) algorithm, the IC-GN algorithm can achieve the same accuracy in less time [17,18].

In terms of accuracy and stability, the IC-GN algorithm has been proven to be mathematically equivalent to the FA-NR algorithm [18]. In addition to accuracy and stability, robustness is an important standard for evaluating algorithms, especially the robustness in the presence of noise [19,20]. The noise robustness of the FA-NR algorithm has been thoroughly investigated by many researchers [15,16]. For example, Wang et al. assessed the quantitative error of the FA-NR algorithm caused by image noise [15]. However, few studies have systematically and quantitatively evaluated the noise robustness of the IC-GN algorithm. Furthermore, which algorithm has better noise robustness is an interesting but confusing question for the users of DIC.

The computation efficiency of IC-GN is still in need of further improvements, although its calculation speed is faster than FA-NR [17,18]. As we know, DIC is a block-by-block processing technique, and parallel computing can help to calculate these blocks concurrently. Recently, a path-independent method combining a fast Fourier transform-based cross correlation (FFT-CC) algorithm and an IC-GN algorithm was proposed [21]. Parallel computing can be achieved based on this method, but the initial guess obtained by the FFT-CC algorithm is limited to translation, and an imperfect initial guess could lead to redundant iterations.

In this paper, a novel one-dimensional (1D) theoretical model of the IC-GN algorithm is proposed. This model, based on the sum of squared differences (SSD) correlation criterion and linear interpolation, clearly indicates that the IC-GN algorithm has better noise robustness than FA-NR algorithm, and shows no noise-induced bias if the gray gradient operator is chosen properly. Both numerical

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http://dx.doi.org/10.1016/j.optlaseng.2015.03.005
0143-8166/

simulations and experiments were conducted to validate this, and the results show good agreement with the theoretical predictions. Moreover, a seed point-based parallel method is proposed to improve the calculation efficiency in Section 4. The implementation and advantages of the proposed method are discussed in detail. The accuracy and computational efficiency of the proposed parallel IC-GN algorithm are demonstrated by real experiments.

2. Brief introduction to the IC-GN algorithm

The IC-GN algorithm was first proposed by Baker and Matthews as an improvement of the Lucas–Kanade algorithm with respect to reducing computing time [18]. The algorithm was initially used as an image alignment method combined with the SSD correlation criterion. Tong [22] suggested using a noise-free reference image as well as additional reference images (generated by offsetting the reference image) in inverse update algorithm to improve its precision. Pan et al. [17] skillfully combined the IC-GN algorithm with the ZNSSD correlation criterion and used it as a sub-pixel DIC registration method. Gao et al. [23] analyzed the performance of IC-GN with first- order and second-order shape functions.

Fig. 1 schematically shows the principles of the IC-GN and FA-NR algorithms, where $W(x,y;p)$ is the shape function that depicts the shape of the target subset relative to the reference subset, x, y are local coordinates, and p is the parameter vector. The practical and robust ZNSSD correlation criterion is used to evaluate the similarity between the reference and target subsets.

In each iteration of the FA-NR algorithm, the incremental warp $W(W(x,y;p);\Delta p)$ is exerted on the target subset, hence the parameter increment Δp can be added to the parameter vector directly (namely, $p^{n+1}=p^n+\Delta p$), n is the number of iterations. However, the incremental warp $W(x,y;\Delta p)$ is first exerted on the reference subset in each iteration of the IC-GN algorithm. Hence, it is subsequently inverted and composed with the deformation of the target subset. If a first-order shape function is used, the operation can be expressed as follows:

$$W(x,y;p) \leftarrow W(x,y;p)\circ W^{-1}(x,y;\Delta p) = \begin{bmatrix} 1+u_x & u_y & u \\ v_x & 1+v_y & v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\Delta u_x & \Delta u_y & \Delta u \\ \Delta v_x & 1+\Delta v_y & \Delta v \\ 0 & 0 & 1 \end{bmatrix}^{-1} \quad (1)$$

where u, v are the displacement components and u_x, u_y, v_x, v_y are the displacement gradients. This iteration is repeated until the pre-set convergence condition is satisfied. In this work, the convergence condition, $\|\Delta p\| \le 0.001$, is predefined.

In IC-GN, the Hessian matrix need not be recalculated after pre-calculation because the reference subset remains the same [18]. Furthermore, there is another outstanding characteristic of IC-GN algorithm, that the first-order Taylor expansion of the grayscale intensities caused by the deformation increment is performed on the integer-pixel points$W(x,y;0)$. Thus, the noise-induced bias error can be suppressed by choosing a proper gray gradient operator, as discussed in Section 3.

3. Noise robustness of IC-GN

For the convenience of analysis, a 1D SSD correlation criterion is used to derive a theoretical model of the IC-GN and FA-NR algorithms. Consider a subset of $2M+1$ pixels, the 1D SSD correlation function is defined as follows:

$$(u)_{opt} = \arg\min \sum_{i=-M}^{M} [f(x_i) - g(x_i+u)]^2 \quad (2)$$

where u is the deformation between the reference and target images, $f(x_i)$ is the gray intensity value at point x_i of the reference image, and $g(x_i+u)$ is the gray intensity value at point x_i+u of the target image.

To obtain the optimal value for u, non-linear iterative algorithms such as FA-NR, IC-GN, Steepest Descent, or Levenberg–Marquardt can be used. FA-NR and IC-GN are the mostly widely used in DIC.

3.1. IC-GN theoretical error model

In IC-GN algorithm, we define u_0 to be the initial value and u' to be the measured translation after iteration. Thus, we have

$$u' = u_0 - \Delta u \quad (3)$$

where Δu is the deformation increment. Here we let u_0 be the same with the exact translation, so the error of the measured translation due to iteration can be obtained.

Minimization of the SSD correlation criterion function with respect to the variable Δu gives

$$\frac{\partial\left(\sum_{i=-M}^{M} [f(x_i+\Delta u) - g(x_i+u_0)]^2\right)}{\partial(\Delta u)} = 0 \quad (4)$$

Here, x_i is the coordinate of an integer-pixel point. Neglecting the high-order terms, the first-order Taylor expansion of $f(x_i+\Delta u)$

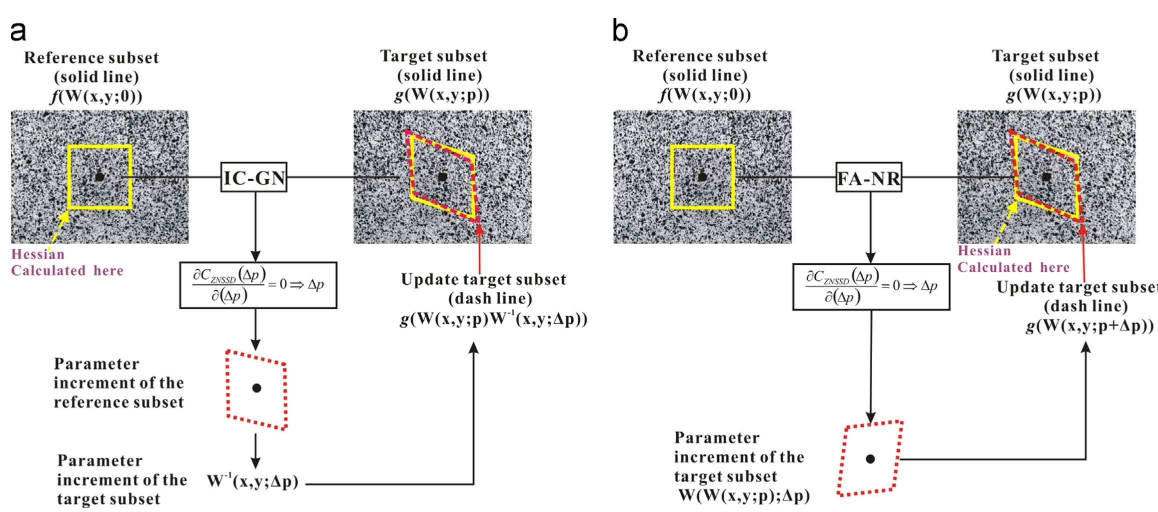

Fig. 1. Schematic of (a) IC-GN and (b) FA-NR.