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Long-term investment with stochastic interest and inflation rates: The need for inflation-indexed bonds[☆]

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ABSTRACT

We examine the long term investment problem, under stochastic interest and inflation rates and within financial market incompleteness. Four basic financial assets are available on the financial market: a money market account (the cash), a real consumption good, a financial stock index and a bond with constant maturity. In this incomplete framework, we provide the general solution of the expected utility maximization. We compute the monetary loss from not having access to an inflation-indexed bond, in order to be hedged against the inflation risk. We show that this latter one usually reaches high levels (more than 1% per year). Thus, the magnitude of such costs reaches those of management fees or transaction costs. They highlight the significant value of introducing inflation-indexed bonds in the financial markets.

1. Introduction

Long term investment is very sensitive to inflation risk. The omission of inflation uncertainty induces several issues when considering long horizons. In this framework, the prediction of real asset returns is a rather involved problem for long time horizons (for example are bonds more safe than equities?).¹ How inflation can modify real portfolio performance? How can we hedge portfolios against inflation? One solution is to include for example precious metals or real estate investments into the portfolio together with standard cash, nominal bonds and equities. Another alternative is to involve inflation linked bonds. Recall that inflation-indexed bonds are bonds the principal of which is indexed to inflation. They are introduced to cut out the inflation risk.

The market of inflation-indexed bonds began to grow when Great Britain issued such financial securities in 1981, the so called inflation-

linked Gilts (ILG). Then other states such as Canada (1991), the US (1997),² France (1998)³ and Germany (2006) introduced also such assets. As emphasized in Laroche and Servel (2014), the inflation-linked bond (ILB) market outstanding has been twice as big as the covered bond market in May 2014. Despite low inflation and low real interest yields, investment on euro-denominated inflation linked bond has notably growth in recent years: In May 2014, Eurozone ILB market represents \$650bn (about 20% of the global ILB market), 25% higher than the 2013 supply over the same period of time. US ILB market represents \$993bn, while the global government inflation-linked debt has reached \$2.85trn. Note that the market for optional instruments on inflation, allowing for example to protect investors if the price increase exceeds a given level, is not developed. Investors prefer their classic indexed bonds as they are considered cheaper and better to really hedge against inflation.

Several authors have studied such specific products and how they

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¹ Rolling positions is also hard, since bonds with very long maturities are not available on financial markets.

² Treasury Inflation Protected Securities (or TIPS) correspond to the inflation-indexed bonds that are issued by the U.S. Treasury. These assets were first issued in 1997. The principal is adjusted to the Consumer Price Index, the commonly used measure of inflation. The coupon rate is constant, but generates a different amount of interest when multiplied by the inflation-adjusted principal, thus protecting the holder against inflation. TIPS are currently offered in 5-year, 10-year and 20-year maturities. 30-year TIPS are no longer offered. In addition to their value for a borrower who desires protection against inflation, TIPS can also be a useful information source for policy makers: By comparing a TIPS bond with a standard nominal Treasury bond across the same maturity dates, investors may calculate the bond market's expected inflation rate by applying the Fisher equation.

³ For instance, in France, there exists a bond indexed on inflation, called OATI, since December 1998.

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can enhance portfolio performance. Lamm (1998) for example deals with the portfolio allocation implications of inflation protection securities. He emphasizes that the investor should hold a zero-investment portfolio of nominal bonds while putting 100% of her wealth in equities and inflation-protected bonds. Campbell and Viceira (2002) prove also that an inflation-indexed perpetuity bond is the riskless asset for infinite-lived investors. They prove that long-term inflation-indexed bonds much increase the utility of conservative investors. Brennan and Xia (2000) show also that such an investor invests in a mix of nominal bonds to replicate the return of an inflation-indexed bond with maturity equal to the remaining investment horizon. Kothari and Shanken (2004) examines the asset allocation problem with stocks, indexed bonds, conventional Treasuries and a riskless asset. They conclude that “substantial weight should be given to indexed bonds in an efficient portfolio.”⁴ Roll (2004) shows that a portfolio diversified between U.S. equities and nominal bonds would be improved by the addition of TIPS. For more specific management periods, the previous results are mitigated. Brière and Signori (2009) use dynamics of conditional volatilities and correlations for three asset classes, indexed bonds, nominal bonds and equities, in the USA and Europe. They emphasize the change that took place in 2003. Due to higher correlations with nominal bonds with similar volatilities, the optimal weight of indexed bonds in a portfolio decreased sharply in 2003. Such conclusion has been also established by Hunter and Simon (2005). However, more recent studies are in favor of inflation-indexed bonds: Results in Illeditsch (2016) are in line with those of Lamm (1998), Cartea et al. (2012) illustrate the benefits of introducing Treasury Inflation-Protected Securities (TIPS) in portfolios. Brière and Signori (2012) conclude that in a volatile macroeconomic framework with countercyclical supply shocks, cash, inflation-linked bonds and precious metals play an essential role, while in a more stable environment with procyclical demand shocks, cash and nominal bonds play the most significant role.

Another problem arises: How can we model inflation rates and calibrate nominal interest rates? Empirical evidence indicates, for example, that the US inflation volatility is smaller than real interest rate volatility (see e.g. Pennacchi, 1991). In this paper, to model the multifactor term structure, we adopt the model introduced by Chiarella et al. (2007). We assume that both interest and inflation rates are stochastic and exhibit mean-reverting returns. Both inflation-indexed and non indexed bonds prices are based on exponential affine models, as introduced by Duffie and Kan (1996). They depend only on two factors: the real interest rate and the inflation rate. In this framework, both inflation-indexed bonds and nominal bonds can be priced. We consider a financial market as in Chiarella et al. (2007), but with different basic tradable assets: we introduce bonds with constant durations. As discussed by Bajoux-Besnainou et al. (BJP) (2001), the introduction of constant maturation bonds allows to obtain a bond/stock ratio which increases with time, when there exists no inflation. This nice property is in accordance with popular advice.⁵ As in BJP (2001), we assume that the investor's horizon exceeds the maturity of the cash asset and they introduce a continuous-time portfolio rebalancing. In that case, cash may be a money market security with a short maturity (one to six months) and may be no longer the common riskless asset in the standard theory. We solve the portfolio optimization problem of an investor who searches to maximize the expected

utility of her real wealth⁶. The portfolio is optimally rebalanced as a function of the remaining time and of current asset values.⁷ Contrary to Mkaouar and Prigent (2016), we assume that inflation-indexed bonds are not available on the financial market, which implies that no perfect hedge exists against the inflation risk.

Our purpose is to measure the consequences of the lack of inflation-indexed bonds on the financial market since these latter ones are assumed to be no more available. In that case, the investor cannot be perfectly hedged against the inflation risk. The financial market becomes incomplete. In this framework, two problems are studied: the first one is the resolution of the optimization problem. For this purpose, first we use the martingale approach introduced by Cvitanic and Karatzas (1992) to solve optimization problems when strategies must respect convex constraints. This allows us to get explicit formula for the optimal portfolio values. The idea is to complete the market with a fictitious asset on which the investor cannot trade. Second, to determine the optimal allocation weights, we use the dynamic programming approach based on Hamilton-Jacobi-Bellman (HJB) method, introduced for the first time by Merton (1971). A well-known property of optimal portfolio is the *mutual fund separation* theorem, which proves that a rational investor divides her investment between two assets: a riskless one and a risky mutual fund, the composition of which is the same, whatever the investor's risk aversion. Cass and Stiglitz (1970) established conditions for two fund-separation, which holds for a large class of utility functions, such as the HARA functions. We show that, in our framework, this result is also true when examining dynamic portfolio strategies with incompleteness due to inflation.

Another consequence of the lack of inflation-indexed bonds is that the new optimal portfolio induces an utility loss when the same initial amount is invested. Several measures can be proposed to quantify the loss of not having access to inflation-indexed bonds. As shown in De Palma and Prigent (2008), De Palma and Prigent (2009), we can introduce either a direct calculation of the utility loss, either a monetary loss based on the concept of *compensating variation*, which is linked to the notion of *certainty equivalence*. We use the latter one to examine the consequence of the lack of the inflation-indexed bonds and illustrate how the corresponding monetary loss depends on risk aversion level.

The paper is organized as follows. Section 2 presents the financial market with its multi factor structure model that describes both the nominal and the inflation-indexed bonds. We determine the risk-neutral probabilities for the complete auxiliary markets, as introduced in Cvitanic and Karatzas (1992). Section 3 provides the solution of the optimization problem for quite general utility functions when the indexed-inflation bond is not available on the market (incomplete case). We detail and illustrate the solution in particular for the logarithmic and CRRA cases. Section 4 examines the consequences of the lack of inflation-indexed bonds, when these later ones are no more available on the financial market. For this purpose, we provide explicit formula for the compensating variation for various risk aversion levels. Some of the proofs are gathered in the appendix.

2. The financial market

We adopt the same framework as in Chiarella et al. (2007) and

⁴ Such results are generally supported by the analysis of the history of actual returns on U.S. Treasury Inflation-Indexed Securities (TIPS) for example from 1997 to 2003.

⁵ Younger investors invest usually a higher fraction of their portfolio value in stocks than older investors. But, as mentioned by Samuelson (1969), this is not consistent with results of basic models of portfolio optimization. Canner et al. (1997) note also that popular investment advice does not conform these results. Additionally, many empirical studies show that allocations between stocks, bonds and cash depend a lot on risk aversion. In particular, bond/stock ratios differ for conservative, moderate or aggressive investors. Bajoux-Besnainou et al. (2001) address this inconsistency issue between mutual fund property and popular advice.

⁶ Recall first that we deal with portfolio optimization in continuous-time and that, for almost all models proposed in the literature about this topic (see e.g. the seminal paper by Merton, 1971), asset prices are assumed to be Markovian. For the standard expected utility criterion, it leads to Markovian strategies. Since our goal is only to illustrate the impact of the introduction of inflation-indexed bonds, we have chosen the most standard framework.

⁷ For simplicity, and in order to obtain analytical solutions, we assume that transaction costs are negligible (which is true for main financial indices). Prices of these assets are also assumed to follow standard diffusion processes.

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