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A generalized CAPM model with asymmetric power distributed errors with an application to portfolio construction

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ABSTRACT

We estimate the CAPM model on European stock market data, allowing for asymmetric and fat-tailed return distributions using independent and identically asymmetric power distributed (IIAPD) innovations. The results indicate that the generalized CAPM with IIAPD errors has desirable properties. It is substantially less likely to be rejected than the traditional CAPM with normally distributed errors and, moreover, backtests show that portfolios constructed using IIAPD errors outperform the portfolio constructed with normally distributed errors in terms of commonly-used performance measures.

1. Introduction

Although the Gaussian (normal) distribution is widely used in capital asset pricing and portfolio selection (see e.g., Markowitz (1952, 1959), Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972), Campbell et al. (1997)), there is ample empirical evidence that returns are usually not normally distributed (see, e.g., Fama (1965), Mandelbrot (1967), Blattberg and Gonedes (1974), Affleck-Graves and McDonald (1989)). According to Campbell et al. (1997), specifically taking asymmetry and fat-tails of financial data into account is relevant for asset pricing.

To cope with fat tails and skewness of financial data, Komunjer (2007) constructs the Asymmetric Power Distribution (APD), extending the Generalized Power Distribution (GPD) by accommodating asymmetry. Likewise, the Asymmetric Exponential Power Distribution (AEPD) proposed by Zhu and Zinde-Walsh (2009) includes two tail parameters to describe the decay of the tail densities. Table 1 indicates how a number of well-known distributions are nested as special cases of the APD and the AEPD.

Despite the vast empirical evidence on non-normality of returns (see Sharpe (2007, 2007)), most existing literature on empirical asset pricing adheres to the assumption of normally distributed returns. In this paper, we consider the seminal Capital Asset Pricing Model (CAPM) under the more general assumption that the error terms are independent and identically asymmetric power distributed (IIAPD) with zero mean, variance σ_ε^2, skewness parameter α, and tail parameter(s) to accommodate the asymmetry and fat tails of the returns distribution. Our work provides a generalization of CAPM because the previously assumed normal return distributions can be considered as special cases of IIAPD. Zeckhauser and Thompson (1970) consider the estimation of linear models with power distributions as highly desirable and expect the effects on the estimated coefficients to be substantial. Thus, considering IIAPD errors potentially is empirically highly relevant to researchers and practitioners.

To examine whether two tail parameters (p_1 and p_2 in the AEPD capture the decay of the left and right tail, respectively) can better describe the distributions of asset returns than one (λ in the APD determines the decay of both tails), we develop two versions of the generalized CAPM with IIAPD errors, namely, the CAPM-IAPD (independent and identically asymmetric power distributed with one tail parameter) and the CAPM-IAEPD (independent and identically asymmetric exponential power distributed with two tail parameters). Using these new models as well as the traditional CAPM, we address the following research questions:

(i) To what extent does the specification of the error term affect whether the CAPM model is 'alive' (working) or 'dead' (rejected)?
(ii) Does the generalized CAPM with non-normal errors outperform

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Table 1
Special cases of the APD and the AEPD.

Distributions	Parameters in APD	Parameters in AEPD
Normal	$\alpha = 1/2, \lambda = 2$	$\alpha = 1/2, p_1 = p_2 = 2$
GPD	$\alpha = 1/2, \lambda > 0$	$\alpha = 1/2, p_1 = p_2$
Laplace	$\alpha = 1/2, \lambda = 1$	$\alpha = 1/2, p_1 = p_2 = 1$
Asymmetric Laplace	$\alpha \neq 1, \lambda = 1$	$\alpha \neq 1/2, p_1 = p_2 = 1$
Two-piece normal	$\alpha \neq 1/2, \lambda = 2$	$\alpha \neq 1/2, p_1 = p_2 = 2$
SEPD		$\alpha \neq 1/2, p_1 = p_2$

The GPD allows flexibility in modeling the tail behavior. According to Komunjer (2007), the GPD corresponds to the uniform distribution when $\lambda = \infty$, short-tailed distributions when $2 \le \lambda < \infty$, and fat-tailed ones when $0 < \lambda < 2$. According to Zhu and Zinde-Walsh (2009), the APD is a sort of SEPD (Skew Exponential Power Distribution) due to the quantification of asymmetry; however, the AEPD corresponds with the SEPD if $p_1 = p_2$.

the traditional CAPM with normal errors in terms of fitness measures?

(iii) Does the generalized CAPM with two tail shape parameters (CAPM-IAEPD) have better fitness and predictive power than the CAMP with only one tail shape parameter (CAPM-IAPD)?

(iv) What does the assumption of IIAPD errors imply for the practitioners in the fields of security valuation and portfolio management? Do the portfolios constructed under this assumption outperform those employing the normality assumption?

Our results show that the specification with IIAPD errors has very nice properties. First, it helps to 'save' the CAPM in the sense that the CAPM is substantially less likely to be rejected with IIAPD errors than with traditional normally distributed errors when using monthly returns data. Second, the generalized CAPM with IIAPD errors outperforms the traditional CAPM in terms of fitness measures in particular with weekly and daily data. Meanwhile, incorporating two parameters for the tail shape does not seem to further improve the fitness or predictive power of the model relative to the model with one parameter for the tail shape.

As indicated in research question (*iv*), we also study what the assumption of IIAPD errors implies for practitioners in the fields of security valuation and portfolio management. Do the portfolios constructed using this assumption outperform those employing the normality assumption? Our results suggest the answer is 'yes'; the portfolios constructed using the IIAPD errors outperform the portfolio employing normal errors, according to the Sharpe Ratios, Jensen's Alphas, and Treynor Ratios.

By assuming the possibility of lending and borrowing at the risk-free rate of interest, the Sharpe-Lintner version of the CAPM is given by

$$\mathbb{E}(R_i) = R_f + \beta_{iM}(\mathbb{E}(R_M) - R_f), \tag{1}$$

where $\mathbb{E}[R_i]$ is the expected return of asset i, R_f is the return on the risk-free asset, and $\mathbb{E}[R_M]$ is the expected return on the market portfolio. The compact form of Eq. (1), in terms of excess returns (i.e. returns in excess of the risk-free rate), is expressed by

$$\mathbb{E}(Z_i) = \beta_{iM}\mathbb{E}(Z_M), \tag{2}$$

where Z_i represents the return on the ith asset in excess of the risk-free rate, $Z_i \equiv R_i - R_f$, Z_M represents the excess return on the market portfolio of assets, $Z_M \equiv R_M - R_f$, and $\beta_{iM} = \frac{\mathrm{Cov}(Z_i, Z_M)}{\mathrm{Var}(Z_M)}$. The data-generating process (DGP) underlying the CAPM is assumed to be given by

$$Z_{it} = \alpha_{iM} + \beta_{iM} Z_{Mt} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathrm{NID}(0, \sigma_{\varepsilon_i}^2), \tag{3}$$

where i denotes the asset, t denotes the time period ($t = 1, \ldots, T$), Z_{it} and Z_{Mt} are the excess returns in time period t for the asset i and the market portfolio, respectively, $Z_{it} \equiv R_{it} - R_{ft}$, $Z_{Mt} \equiv R_{Mt} - R_{ft}$, α_{iM} represents the asset excess return intercept, β_{iM} is the proportionality factor that reflects the sensitivity of asset i relative to the market risk, and ε_{it} is a normally, independent distributed (NID) random variable with mean 0 and variance $\sigma_{\varepsilon_i}^2$.

To examine the effects of the assumption of IIAPD innovations on the estimated coefficient, we consider Question (*i*). As in several previous papers (see e.g., Fama and French (1996)), here 'alive' implies that α_{iM} is statistically insignificant and β_{iM} is statistically significant; thus, if at least one of them does not hold, the CAPM is called 'dead'. Therefore, Question (*i*) will be studied via the 95% confidence intervals of α_{iM} and β_{iM}. In addition, the Akaike Information Criterion (AIC) and Bayesian information criterion (BIC), density plots of the residuals, and distribution tests are used to address Questions (*ii*) and (*iii*). Finally, an applicable portfolio strategy, the Minimum Variance Portfolio (MVP), will be presented in Section 6 with backtesting based on historical data. The MVP is applied because its construction does not require the estimation of expected returns, which are difficult to estimate, and the optimization of MVP depends on the estimates of β_{iM}. Moreover, it performs well in practice and is popular among investors.

The empirical analysis is conducted on the European market. The EURO STOXX 50 index is used as a proxy of the market portfolio. Maximum Likelihood is used to analyze its constituents. We estimate the CAPM using daily, weekly as well as monthly returns. After that, backtesting on a monthly basis is used to investigate portfolio construction strategies.

To our knowledge, there is very little previous work in which IIAPD errors are applied to model asset returns conditional on market portfolios. Li and Lin (2014) checked the validity of CAPM for the French stock market based on a model called CAPM-AEPD, however, they used the location parameter μ and the scale parameter σ of the AEPD as the proxies of the mean and standard deviation of the distribution. Although μ and σ indeed correspond to the mean and standard deviation in the special case of a normal distribution, they are not suitable proxies for the mean and standard deviation of the APD or AEPD.

Our paper is also related to a broad class of studies on the application of other non-Gaussian distributions in finance. The class of symmetric stable Paretian distributions has, for instance, been used to model fat-tailed return distributions (see e.g., Mandelbrot (1963, 1963); Fama (1965, 1971)). In addition, portfolio theory under stable Paretian laws has been developed (see Rachev and Mittnik (2000)). However, Cootner (1964) argues that the evidence of Paretian distributed security returns is too casual. Campbell et al. (1997) deem that the stable Paretian distributions are too fat-tailed. Moreover, Thomas and Gup (2010) argue that the empirical patterns of stock returns are inconsistent with the stable Paretian theory of constant 'alpha peakedness' and 'beta skewness' for returns over different time intervals. As a result, the stable Paretian does not appear to be the ideal distribution to model the fat tails of financial returns.

A number of alternative distributions have been proposed in the literature. For the sake of flexibility in modeling fat tails, the seminal Exponential Power Distribution (EPD, also called the Generalized Laplace distribution, the Generalized Power Distribution, or the Generalized Error Distribution) family proposed by Subbotin (1923) has also been applied in finance (see, e.g., Harvey (1981); Nelson (1991)). However, the EPD does not allow for asymmetry in financial returns data. The Skew Normal (SN) distribution and the Skew Exponential Power distribution (SEPD) proposed by Azzalini (1985, 1986) can fit data with skew distributions. Despite this attractive property, the skewness parameter may be estimated to be infinite. Moreover, consistency of the maximum likelihood estimators (MLEs) has not been established for the SN distribution and the SEPD. In contrast, Fernández et al. (1995); Theodossiou (2000) and Komunjer (2007) extend the EPD family in the sense of accommodating asymmetry, categorized by Zhu and Zinde-Walsh (2009) as the second method of constructing SEPD. For the second method of constructing SEPD family, Zhu and Zinde-Walsh (2009) prove consistency of MLEs when the tail shape parameter is larger than one. Besides, Komunjer