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Sensitivity of singular beams in the presence of Zernike aberrations

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ABSTRACT

Singular beams in the presence of Zernike aberrations create an opportunity for various applications such as trapping and manipulation of micro-particles, atomic optics and atmospheric optics. In the milieu of importance of the role of aberrations, sensitivity of singular beams with Zernike aberrations is studied. In this paper, the effect of various Zernike aberrations on a singular beam is reported in terms of its Point Spread Function (PSF) deformations. The intensity distributions around the focal plane, i.e. PSF, of the singular beam of various topological charges and in the presence of different strengths of Zernike aberrations are theoretically estimated by the Huygens–Fresnel diffraction integral. Experimentally, the singular beams have been generated and known strengths of Zernike aberration introduced in the beam by a phase-only Spatial Light Modulator. Metric Ensquared Energy is used to analyze the PSF of the corresponding intensity distributions of the singular beams. The experimental results have been validated with numerical simulation.

1. Introduction

It is well known that laser beam propagating through atmosphere suffers from intensity as well as phase fluctuations. Under strong turbulence conditions, scintillation (i.e. Intensity fluctuation) arises in the laser beam when turbulence induced phase fluctuations in one region are converted into intensity fluctuations while propagation over long distances [1]. This intensity fluctuation results in the evolution of singular phases in the wavefront. Laser beam having phase singularity with topological charge, m located anywhere in the wavefront is considered as a singular beam [2]. Singular beam is less perturbed by the atmosphere while propagating through the atmosphere [3]. During the propagation through the turbulence, optical vortices in the beam can be created and destroyed in such a way that the total topological charge remains conserved. Wang et.al, have investigated theoretically that coherent vortex beams are less influenced by the atmospheric turbulence than partially coherent non-vortex beams, and the higher the topological charge of the beam, lesser is the influence of atmospheric turbulence on the beam [3]. They have also demonstrated that the beam spreading due to turbulence in the vortex beam is smaller in comparison to Gaussian beam and also experimentally investigated that the topological charges exhibit fluctuating behavior during propagation through atmosphere [4]. Lukin et.al, have recently analyzed the beam spreading and found that the vortex beam gets less broadened than a Gaussian beam while propagating through a randomly inhomogeneous medium. They have also shown that the higher the topological charge, the smaller the beam spreading [5]. Phase fluctuations (i.e. wavefront errors) induced by the atmospheric turbulence in any laser beam can be expressed in terms of the combinations of various Zernike aberrations [6]. So, the effects of individual Zernike aberrations on the singular beam are studied.

Singular beam has points at which phase is undetermined and amplitude are zero; it has a unique characteristic of dark hole in the diffraction pattern and wavefront has helical structure. These beams are used in various applications like trapping and manipulation of micro particles [7], digital holography [8], optical testing [9], quantum cryptography [10], image processing [11], signal processing and astronomy [12], radar imaging [13] and long range free space optical/quantum communications [14]. Singular beam can be generated using a spiral phase plate [15], helical mirror [16], deformable mirror [17], wedge plates [18], customized wavefront tilts [19], phase-only diffractive optical element [20] and the computer generated hologram (CGH) [21,22] in the laboratory.

The intensity distribution of singular beam around the focal plane in the presence of various Zernike aberrations such as defocus, astigmatism, coma and spherical aberration has been studied theoretically [23–28]. Singh et.al, numerically computed the intensity distribution of singular beam in the presence of spherical aberration and concluded that the intensity maximum is decreased with increase in the size of dark hole and the effect of

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spherical aberration is compensated by appropriate value of defocus aberration [23]. Luo et.al, explained the evolution behavior of phase singularities in the presence of spherical aberration and for controlling the phase singularities [24]. Singh et.al, studied the behavior of singular beam in the presence of coma aberrations, PSF split and shift in the vortex hole for topological charge $m=1$, for a higher order beam PSF splits into more parts depending on the topological charge [25]. Singh et.al, computed the intensity distribution of singular beam in the presence of astigmatism aberration numerically and showed that PSF is elongated and transformed into the tail shape, for higher topological charge PSF splits in many parts separated by dark regions [26]. Zhang et.al, studied the spatial distribution of phase singularities in the presence of astigmatism aberration and for controlling the phase singularities [27]. To the best of our knowledge no one has experimentally validated the effect of Zernike aberrations on the singular beams. We report the experimental results along with the numerical simulation.

In this paper, we have comprehensively investigated the behavior of singular beams having phase singularity at the center in the presence of various Zernike aberrations of quantified strength. The intensity distribution around the focal plane and the corresponding Ensquared Energy of the singular beams are estimated numerically using the Huygens–Fresnel diffraction integral for studying the effect of Zernike aberrations such as defocus, spherical, astigmatism, coma and trefoil aberrations. Theoretical results are compared with those of other researchers and found to be in good agreement. Experimentally, singular beams of various topological charges are generated and the effects of Zernike aberrations on them are studied by analyzing the intensity distribution around the focal plane. Here, singular beams are generated using a CGH (which is displayed on phase-only Spatial Light Modulator). In addition to the theoretical work done by other researchers, we have also investigated the effect of trefoil aberration on the singular beam. Thus, in this paper we have presented the study up to third order Zernike aberrations.

2. Theoretical model

Optical beam carrying phase singularity is represented by the function $U(\rho,\theta)$ (Eq. (1)), but in the presence of Zernike aberrations, function $U\ (\rho,\theta)$ is modified and given as in Eq. (2).

$$U(\rho,\theta)=U_o\ (\rho,\theta)e^{(im\theta)} \tag{1}$$

$$U(\rho,\theta)=U_o\ (\rho,\theta)e^{(im\theta)}e^{[\{ikW(\rho,\theta)\}]} \tag{2}$$

$$\begin{aligned}W(\rho,\theta)=&A_d(2\rho^2-1)+A_a\rho^2\ \cos\ 2\theta+A_c((3\rho^3-2\rho)\cos\ \theta\\&+A_t\rho^3\ \cos\ 3\theta+A_s(6\rho^4-6\rho^2+1)\end{aligned} \tag{3}$$

where (ρ,θ) and $(r,ø)$ are the polar coordinates in the pupil plane and the focal plane respectively, U_o is a constant amplitude, m is the topological charge located at the center of the beam and $W(\rho,\theta)$ is the aberration function. Optical aberrations can be expressed as combinations of various Zernike aberrations [29]. A_d, A_s, A_a, A_c and A_t are the coefficients representing the strengths of defocus, spherical, astigmatism, coma and trefoil aberrations, respectively. In this paper, the effects of Zernike aberrations are studied on a phase singularity with topological charge m located at the center of the wavefront.

The Huygens–Fresnel diffraction integral is used to estimate intensity distribution at the focal plane [30]. The geometry illustrating the coordinate system for diffraction is shown in Fig. 1. The far-field intensity distribution is evaluated by the Huygens–Fresnel diffraction integral and is given as

$$U(r,\phi)=\frac{e^{ikz}e^{i(kr^2/2z)}}{i\lambda z}\int_0^{2\pi}\int_A U(\rho,\theta)e^{-i(kr\rho/z)\ \cos\ (\phi-\theta)}\rho d\rho d\theta \tag{4}$$

In Cartesian coordinates, the equation is written as

$$U(x,y,z)=\frac{e^{ikz}}{i\lambda z}e^{i(k(x^2+y^2)/2z)}\iint_A U(x_0,y_0,0)e^{-i(k/z)\ (x_0x+y_0y)}dx_0dy_0 \tag{5}$$

where λ is the wavelength of the light source, z is the propagation distance and k is wave number, i.e., equal to $2\pi/\lambda$.

Intensity distribution at the focal plane is then given by the following equation:

$$E(x,y,z)=|U(x,y,z)|^2 \tag{6}$$

Here, the intensity distribution refers to the far-field image of a point object. Hence the intensity distribution is the same as Point Spread Function (PSF). In the intensity profiles, all the intensities are normalized with respect to the singular beam with no aberrations.

Encircled Energy is the important corollary of the PSF. It is one of the significant parameters which serve as an index of the performance of an optical system. The Encircled Energy (EcE) measures the fraction of the total energy in the PSF, which lies within a specified radius (τ) in the plane of observation. Encircled Energy is given by the following equation:

$$EcE(\tau)=\frac{\int_0^{2\pi}\int_0^{\tau}E(\tau,\phi)\tau d r d\phi}{\int_0^{2\pi}\int_0^{A}E(\tau,\phi)\tau d r d\phi} \tag{7}$$

where A is the maximum diameter of the area which is under evaluation. Since detectors have square pixels, it is often more convenient to evaluate the energy falling within a certain number of pixels (Ensquared Energy) instead of the encircled energy, which requires interpolation to account for the fractional pixels intercepted by a circular aperture. The Ensquared Energy (EsE) is the fraction of total energy contained within a square of given

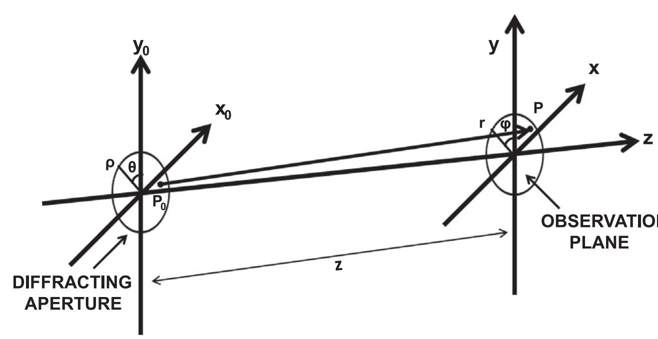

Fig. 1. Geometry illustrating the coordinate system for diffraction.

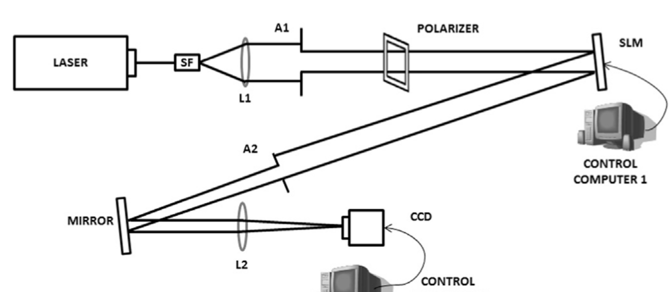

Fig. 2. Experimental schematic for generating the various order singular beams and recording the intensity distribution around the focal plane.